

Phase Shift

$$\begin{aligned}\sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos(\theta) \\ \tan(-\theta) &= -\tan \theta\end{aligned}$$

$$\begin{aligned}\cot(-\theta) &= -\cot(\theta) \\ \sec(-\theta) &= \sec \theta \\ \csc(-\theta) &= -\csc(\theta)\end{aligned}$$

$$\begin{aligned}\sin(2\pi + \theta) &= \sin \theta \\ \cos(2\pi + \theta) &= \cos \theta \\ \tan(2\pi + \theta) &= \tan \theta\end{aligned}$$

$$\begin{aligned}\sin(2\pi - \theta) &= -\sin \theta \\ \cos(2\pi - \theta) &= \cos \theta \\ \tan(2\pi - \theta) &= -\tan \theta\end{aligned}$$

$$\begin{aligned}\sin(\theta - 2\pi) &= \sin \theta \\ \cos(\theta - 2\pi) &= \cos \theta \\ \tan(\theta - 2\pi) &= \tan \theta\end{aligned}$$

$$\begin{aligned}\cot(2\pi + \theta) &= \cot \theta \\ \sec(2\pi + \theta) &= \sec \theta \\ \csc(2\pi + \theta) &= \csc \theta\end{aligned}$$

$$\begin{aligned}\cot(2\pi - \theta) &= -\cot \theta \\ \sec(2\pi - \theta) &= \sec \theta \\ \csc(2\pi - \theta) &= -\csc \theta\end{aligned}$$

$$\begin{aligned}\cot(\theta - 2\pi) &= \cot \theta \\ \sec(\theta - 2\pi) &= \sec \theta \\ \csc(\theta - 2\pi) &= \csc \theta\end{aligned}$$

$$\begin{aligned}\sin(\pi + \theta) &= -\sin \theta \\ \cos(\pi + \theta) &= -\cos \theta \\ \tan(\pi + \theta) &= \tan \theta\end{aligned}$$

$$\begin{aligned}\sin(\pi - \theta) &= \sin \theta \\ \cos(\pi - \theta) &= -\cos \theta \\ \tan(\pi - \theta) &= -\tan \theta\end{aligned}$$

$$\begin{aligned}\sin(\theta - \pi) &= -\sin \theta \\ \cos(\theta - \pi) &= -\cos \theta \\ \tan(\theta - \pi) &= \tan \theta\end{aligned}$$

$$\begin{aligned}\cot(\pi + \theta) &= \cot \theta \\ \sec(\pi + \theta) &= -\sec \theta \\ \csc(\pi + \theta) &= -\csc \theta\end{aligned}$$

$$\begin{aligned}\cot(\pi - \theta) &= -\cot \theta \\ \sec(\pi - \theta) &= -\sec \theta \\ \csc(\pi - \theta) &= \csc \theta\end{aligned}$$

$$\begin{aligned}\cot(\theta - \pi) &= \cot \theta \\ \sec(\theta - \pi) &= -\sec \theta \\ \csc(\theta - \pi) &= -\csc \theta\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi}{2} + \theta\right) &= \cos \theta \\ \cos\left(\frac{\pi}{2} + \theta\right) &= -\sin \theta \\ \tan\left(\frac{\pi}{2} + \theta\right) &= -\cot \theta\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta\end{aligned}$$

$$\begin{aligned}\sin\left(\theta - \frac{\pi}{2}\right) &= -\cos \theta \\ \cos\left(\theta - \frac{\pi}{2}\right) &= \sin \theta \\ \tan\left(\theta - \frac{\pi}{2}\right) &= -\cot \theta\end{aligned}$$

$$\begin{aligned}\cot\left(\frac{\pi}{2} + \theta\right) &= -\tan \theta \\ \sec\left(\frac{\pi}{2} + \theta\right) &= -\csc \theta \\ \csc\left(\frac{\pi}{2} + \theta\right) &= \sec \theta\end{aligned}$$

$$\begin{aligned}\cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta \\ \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta \\ \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta\end{aligned}$$

$$\begin{aligned}\cot\left(\theta - \frac{\pi}{2}\right) &= -\tan \theta \\ \sec\left(\theta - \frac{\pi}{2}\right) &= \csc \theta \\ \csc\left(\theta - \frac{\pi}{2}\right) &= -\sec \theta\end{aligned}$$

Solution for $\sin x = a$ where $a \neq \pm 1$:

$$x = n\pi + (-1)^n \sin^{-1} a$$

$$(\sin^{-1} a \in [-\frac{\pi}{2}, \frac{\pi}{2}])$$

Solution for $\sin x = 1$: $x = (2n + \frac{1}{2})\pi$

Solution for $\sin x = -1$: $x = (2n - \frac{1}{2})\pi$

Solution for $\cos x = a$ where $a \neq \pm 1$:

$$x = 2n\pi \pm \cos^{-1} a$$

$$(\cos^{-1} a \in [0, \pi])$$

Solution for $\cos x = 1$: $x = 2n\pi$

Solution for $\cos x = -1$: $x = (2n + 1)\pi$

Solution for $\tan x = a$:

$$x = n\pi + \tan^{-1} a$$

$$(\tan^{-1} a \in \mathbb{R})$$