

Co-ordinate Geometry

In the following, $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given two different points.

$$\text{Midpoint } M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Internal Division

M is on PQ , $\overrightarrow{PM} : \overrightarrow{MQ} = k : l$ where $k > 0$ and $l > 0$.

$$M = \left(\frac{kx_2 + lx_1}{k + l}, \frac{ky_2 + ly_1}{k + l} \right)$$

If $k < l$, M is closer to P . If $k > l$, M is closer to Q .

If $k = l$, M becomes the midpoint of PQ .

External Division

M is *not* on PQ , $\overrightarrow{PM} : \overrightarrow{MQ} = k : l$ where $kl < 0$ and $|k| \neq |l|$.

$$M = \left(\frac{kx_2 + lx_1}{k + l}, \frac{ky_2 + ly_1}{k + l} \right)$$

If $|k| < |l|$, M is on the side of P . If $|k| > |l|$, M is on the side of Q .

$$\text{Distance between } P \text{ and } Q : d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Gradient of } \overrightarrow{PQ} : m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$