

Absolute Values

Definition: $|x| = \begin{cases} x, & \text{for } x \geq 0, \\ -x, & \text{for } x < 0 \end{cases}$

Properties:

$$|x| \geq 0$$

$$|x| = \sqrt{x^2}$$

$$-|x| \leq x \leq |x|$$

$$|x| - |y| \leq |x + y| \leq |x| + |y|$$

$$|-x| = |x|$$

$$|x - y| = |y - x|$$

$$|xy| = |x| |y|$$

$$\left| \frac{x}{y} \right| = \frac{|x|}{|y|}, \text{ where } y \neq 0$$

Equations and Inequalities:

For any real number $a \geq 0$,

If $|f(x)| = a$, then $f(x) = a$ or $f(x) = -a$

If $|f(x)| < a$, then $-a < f(x) < a$

If $|f(x)| > a$, then $f(x) > a$ or $f(x) < -a$