Motion Basics

Notations: x Displacement (different from distance travelled)

$\dot{x}$ Velocity (directional - one dimension only in this context); $v = \dot{x} = \frac{dx}{dt}$

$\ddot{x}$ Acceleration (change of Velocity - directional as well); $a = \ddot{x} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

It is generally assumed that when $t = 0, x(0) = 0$ and $v(0) = u$ unless stated otherwise. It follows that

$$\big[x(t)\big]_0^x = x = \int_0^t v(t)\,dt \qquad \text{and} \qquad \big[v(t)\big]_u^v = v - u = \int_0^t a(t)\,dt$$

Other "unconventional" notations or assumptions:

Overloading of the same symbol as dummy variables:

e.g. In $x = \displaystyle\int_0^t v(t)\,dt$, the t in $\displaystyle\int_0^t$ is the parameter of function $x(t)$,

while the t in $v(t)\,dt$ is a dummy variable for the integration form.

(But writing $x = \int_0^t v(u)\,du$ would be even more confusing.)

Same measurement but different parameters:

e.g. $v(t)$ is velocity in terms of time, while $v(x)$ is velocity at a position.

Simple Facts

Either $u \neq 0$ or $\ddot{x} \neq 0$ is required to kick start the motion.

$v = 0$ and $\ddot{x} = 0$ means the particle remain stationary, which is different from being momentarily stationary when $v = 0$ but $\ddot{x} \neq 0$.

Notes on Physics: $F = ma = m\ddot{x}$ (Newton's Second Law) is the only formula of Physics involved in this context.

(Newton's First Law: No force, no acceleration. The particle remains stationary or travel at constant speed along a straight line.)

All other relations will be given in terms of $x, \dot{x}, \ddot{x}$ and t. Some may have physical meaning (e.g. $a(x)$ is a force field), while others in this context may not reflect how the nature works.

Some measurements are encapsulated in constants to suit this level of mathematics.

However, mass m is usually left out so that it can be cancelled in $F = ma$. e.g. $R = mkv$

"One on One": In many motion problems, it is required to express x, v, a and t in terms of one another.

Find a from $v(x)$:

$$\boxed{\ddot{x} = v \cdot \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)} \qquad \ddot{x} = \frac{dv}{dt} = \frac{dx}{dt} \cdot \frac{dv}{dx} = v \cdot \frac{dv}{dx}$$

Find v^2 from $a(x)$:

$$a(x) = \frac{d}{dx}\left(\frac{1}{2}v^2\right), \quad \int_0^x a(x)\,dx = \left[\frac{1}{2}v^2\right]_u^v, \quad v^2 = u^2 + 2\int_0^x a(x)\,dx$$

Alternatively,

$$a(x) = v \cdot \frac{dv}{dx}, \quad \int_0^x a(x)\,dx = \int_0^x v \cdot \frac{dv}{dx}\,dx = \int_u^v v\,dv = \left[\frac{1}{2}v^2\right]_u^v, \quad v^2 = u^2 + 2\int_0^x f(x)\,dx$$

Find v from $a(v)$:

$$\frac{dv}{dt} = a(v), \quad \frac{dt}{dv} = \frac{1}{a(v)}, \quad t = \int_u^v \frac{dv}{a(v)}, \quad \ldots$$

Find x from $a(v)$:

$$a(v) = v \cdot \frac{dv}{dx}, \quad \frac{dx}{dv} = \frac{v}{a(v)}, \quad x = \int_u^v \frac{v}{a(v)}\,dv, \quad \ldots$$