Circular Motion

Basic Equations: Tangental Force $F_T = m\,a_T$, Norminal (Radial) Force $F_N = m\,a_N$.

Let l be the arc length travelled, and θ be the angle such arc spans, then $l = r\theta$.

$v = \dfrac{dl}{dt} = r\dfrac{d\theta}{dt} = r\dot{\theta}$. Angular velocity is defined as $\boxed{\omega = \dot{\theta}}$. $\therefore \boxed{v = r\omega}$.

$a_T = \dot{v} = r\dot{\omega} \quad \ldots (1)$

Projections on x-axis and y-axis:

$$\boxed{x = r\cos\theta}$$

$$\dot{x} = -r\sin\theta\cdot\dot{\theta} \quad \therefore \boxed{\dot{x} = -r\omega\sin\theta = -v\sin\theta}$$

$$\ddot{x} = -r\sin\theta\cdot\dot{\omega} - r\omega\cos\theta\cdot\dot{\theta} \quad \therefore \boxed{\ddot{x} = -r\dot{\omega}\sin\theta - r\omega^2\cos\theta}$$

$$\boxed{y = r\sin\theta}$$

$$\dot{y} = r\cos\theta\cdot\dot{\theta} \quad \therefore \boxed{\dot{y} = r\omega\cos\theta = v\cos\theta}$$

$$\ddot{y} = r\cos\theta\cdot\dot{\omega} - r\omega\sin\theta\cdot\dot{\theta} \quad \therefore \boxed{\ddot{y} = r\dot{\omega}\cos\theta - r\omega^2\sin\theta}$$

Acceleration: Let a be the magnitude of acceleration.

$$\begin{aligned}
a^2 &= \ddot{x}^2 + \ddot{y}^2 \\
&= (-r\dot{\omega}\sin\theta - r\omega^2\cos\theta)^2 + (r\dot{\omega}\cos\theta - r\omega^2\sin\theta)^2 \\
&= r^2\left[(\dot{\omega}^2\sin^2\theta + 2\dot{\omega}\sin\theta\cdot\omega^2\cos\theta + \omega^4\cos^2\theta) + (\dot{\omega}^2\cos^2\theta - 2\dot{\omega}\cos\theta\cdot\omega^2\sin\theta + \omega^4\sin^2\theta)\right] \\
&= r^2\left[\dot{\omega}^2(\sin^2\theta + \cos^2\theta) + \omega^4(\sin^2\theta + \cos^2\theta)\right] \\
&= r^2(\dot{\omega}^2 + \omega^4)
\end{aligned}$$

$$\therefore \quad \boxed{a = r\sqrt{\dot{\omega}^2 + \omega^4}}$$

$\boxed{a_T = r\dot{\omega}}$ From (1)

$a^2 = {a_N}^2 + {a_T}^2$, ${a_N}^2 = a^2 - {a_T}^2 = r^2(\dot{\omega}^2 + \omega^4) - r^2\dot{\omega}^2 = r^2\omega^4$,

$$\therefore \quad \boxed{a_N = r\omega^2}$$

As a result,

$$\boxed{\ddot{x} = -a_T\sin\theta - a_N\cos\theta}$$

$$\boxed{\ddot{y} = a_T\cos\theta - a_N\sin\theta}$$

To obtain the direction of a: $\tan\phi = \dfrac{\ddot{y}}{\ddot{x}} = \dfrac{a_T\cos\theta - a_N\sin\theta}{-a_T\sin\theta - a_N\cos\theta} \div \dfrac{-a_N\cos\theta}{-a_N\cos\theta} = \dfrac{-\frac{a_T}{a_N} + \tan\theta}{\frac{a_T}{a_N}\tan\theta + 1}$

$\therefore \tan\phi = \tan(\theta - \psi)$, where $\psi = \dfrac{a_T}{a_N}$

Uniform Circular Motion : v is constant, so is ω . $\therefore \dot{\omega} = 0$. Substitute into the general formulae ...

$\boxed{a_T = 0}$, $\boxed{a = a_N = r\omega^2}$

$\boxed{\ddot{x} = -r\omega^2\cos\theta = -a_N\cos\theta}$, $\boxed{\ddot{y} = -r\omega^2\sin\theta - a_N\sin\theta}$

Frequency f – the number of revolutions (θ increased by 2π) per second: $\boxed{f = \dfrac{\omega}{2\pi} \quad \text{sec}^{-1}}$

Period T – the time for a revolution: $\boxed{T = \dfrac{1 \text{ sec}}{f} = \dfrac{2\pi}{\omega} \quad \text{sec}}$