The Shift

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \tag{1}$$
$$\int_{\frac{a}{2}}^a f(x)\,dx = \int_0^{\frac{a}{2}} f(a-x)\,dx \tag{2}$$
$$\int_0^{\frac{a}{2}} f(x)\,dx = \int_{\frac{a}{2}}^a f(a-x)\,dx \tag{3}$$
$$\int_0^a f(x)\,dx = \int_0^{\frac{a}{2}} f(x)+f(a-x)\,dx \tag{4}$$
$$\int_0^a f(x)\,dx = \int_{\frac{a}{2}}^a f(x)+f(a-x)\,dx \tag{5}$$

$$\text{If } f(x)+f(a-x) = f(a)\,,$$
$$\int_0^a f(x)\,dx = \frac{a}{2}f(a)$$

Proof:
Let $u = a-x\,,\quad x = a-u\,,\quad dx = -du$

(1):

$$\int_0^a f(x)\,dx = -\int_{a-0}^{a-a} f(a-u)\,du = \int_0^a f(a-x)\,dx$$

(2):

$$\int_{\frac{a}{2}}^a f(x)\,dx = -\int_{a-\frac{a}{2}}^{a-a} f(a-u)\,du = \int_0^{\frac{a}{2}} f(a-x)\,dx$$

(3):

$$\int_0^{\frac{a}{2}} f(x)\,dx = -\int_{a-0}^{a-\frac{a}{2}} f(a-u)\,du = \int_{\frac{a}{2}}^a f(a-x)\,dx$$

(4):

$$\int_0^a f(x)\,dx = \int_0^{\frac{a}{2}} f(x)\,dx + \int_{\frac{a}{2}}^a f(x)\,dx$$
$$= \int_0^{\frac{a}{2}} f(x)\,dx + \int_0^{\frac{a}{2}} f(a-x)\,dx = \int_0^{\frac{a}{2}} f(x)+f(a-x)\,dx$$

(5):

$$\int_0^a f(x)\,dx = \int_0^{\frac{a}{2}} f(x)+f(a-x)\,dx$$
$$= \int_{\frac{a}{2}}^a f(a-x)+f(a-(a-x))\,dx$$
$$= \int_{\frac{a}{2}}^a f(x)+f(a-x)\,dx$$