Table of Integratls - More Trigonometric Functions

$$\int \frac{dx}{1 \pm \sin x} = \frac{-2}{\tan \frac{x}{2} \pm 1} + C_1 = \frac{2}{\cot \frac{x}{2} \pm 1} + C_2 \qquad \left(\frac{2}{\cot \frac{x}{2} \pm 1} - \frac{-2}{\tan \frac{x}{2} \pm 1} = \pm 2 \right)$$

$$\int \frac{dx}{a \pm b \sin x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{a \tan \frac{x}{2} \pm b}{\sqrt{a^2 - b^2}} \right) + C \text{ , where } a > b \geq 0$$

$$\int \frac{dx}{a \pm b \sin x} = \frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{a \tan \frac{x}{2} \pm b - \sqrt{b^2 - a^2}}{a \tan \frac{x}{2} \pm b + \sqrt{b^2 - a^2}} \right| + C_1 \text{ , where } b > a > 0$$

$$= -\frac{2}{\sqrt{b^2 - a^2}} \tanh^{-1} \left(\frac{a \tan \frac{x}{2} \pm b}{\sqrt{b^2 - a^2}} \right) + C_2$$

$$\int \frac{dx}{\sin x} = \ln \left| \tan \frac{x}{2} \right| + C \quad \text{(the case of } a = 0)$$

$$\int \frac{dx}{1 + \cos x} = \tan \frac{x}{2} + C$$

$$\int \frac{dx}{1 - \cos x} = -\cot \frac{x}{2} + C$$

$$\int \frac{dx}{a \pm b \cos x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left[\frac{(a \mp b) \tan \frac{x}{2}}{\sqrt{a^2 - b^2}} \right] + C \text{ , where } a > b \geq 0$$

$$\int \frac{dx}{\pm a + b \cos x} = \frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{(b \mp a) \tan \frac{x}{2} + \sqrt{b^2 - a^2}}{(b \mp a) \tan \frac{x}{2} - \sqrt{b^2 - a^2}} \right| + C_1 \text{ , where } b > a \geq 0$$

$$= \frac{2}{\sqrt{b^2 - a^2}} \tanh^{-1} \left[\frac{(b \mp a) \tan \frac{x}{2}}{\sqrt{b^2 - a^2}} \right] + C_2$$

$$\int \frac{dx}{a \cos x + b \sin x} = \frac{1}{\sqrt{a^2 + b^2}} \ln \left| \frac{\sqrt{a^2 + b^2} + a \tan \frac{x}{2} - b}{\sqrt{a^2 + b^2} - a \tan \frac{x}{2} + b} \right| + C_1 \text{ , where } a > 0$$

$$= \frac{2}{\sqrt{a^2 + b^2}} \tanh^{-1} \left(\frac{a \tan \frac{x}{2} - b}{\sqrt{a^2 + b^2}} \right) + C_2$$

$$\int \frac{dx}{a \cos x + b \sin x + c} = -\frac{2}{(c - a) \tan \frac{x}{2} + b} + C \text{ , where } c^2 = a^2 + b^2$$

$$\int \frac{dx}{a \cos x + b \sin x + c} = \frac{2}{\sqrt{c^2 - a^2 - b^2}} \tan^{-1} \left[\frac{(c - a) \tan \frac{x}{2} + b}{\sqrt{c^2 - a^2 - b^2}} \right] + C \text{ , where } c^2 > a^2 + b^2$$

$$\int \frac{dx}{a \cos x + b \sin x + c} = \frac{1}{\sqrt{a^2 + b^2 - c^2}} \ln \left| \frac{(c - a) \tan \frac{x}{2} + b - \sqrt{a^2 + b^2 - c^2}}{(c - a) \tan \frac{x}{2} + b + \sqrt{a^2 + b^2 - c^2}} \right| + C \text{ , where } c^2 < a^2 + b^2$$

$$\left(\text{Note : } \quad \tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x} \text{ , where } |x| < 1 \right)$$