

## Integration by Parts

**The 'uv' form:** 
$$\int u \, dv = uv - \int v \, du$$

Illustration: 
$$\begin{aligned} \frac{d}{dx}(uv) &= u \frac{dv}{dx} + v \frac{du}{dx} \\ \int \left[ \frac{d}{dx}(uv) \right] dx &= \int \left[ u \frac{dv}{dx} + v \frac{du}{dx} \right] dx \\ uv &= \int \left[ u \frac{dv}{dx} \right] dx + \int \left[ v \frac{du}{dx} \right] dx \\ uv &= \int u \, dv + \int v \, du \\ \int u \, dv &= uv - \int v \, du \end{aligned}$$

**The 'f(x) g(x)' form:** 
$$\int f(x) g'(x) \, dx = f(x) g(x) - \int g(x) f'(x) \, dx$$

(You may consider  $u = f(x)$ ,  $v = g(x)$ ,  $du = f'(x) \, dx$ ,  $dv = g'(x) \, dx$ .)

Illustration: 
$$\begin{aligned} [f(x) g(x)]' &= f(x) g'(x) + g(x) f'(x) \\ f(x) g(x) &= \int f(x) g'(x) \, dx + \int g(x) f'(x) \, dx \\ \int f(x) g'(x) \, dx &= f(x) g(x) - \int g(x) f'(x) \, dx \end{aligned}$$