Limits

In the following, $\lim\limits_{x\to a} f(x) = F$ and $\lim\limits_{x\to a} g(x) = G$.

$$\lim_{x\to a} kf(x) = k \times \lim_{x\to a} f(x) = kF$$

$$\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = F \pm G$$

$$\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x) = F \cdot G$$

$$\lim_{x\to a} [f(x)]^n = [F]^n$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \frac{F}{G}\,, \quad \text{where } G \neq 0\,.$$

$$\lim_{x\to a} \left[e^{f(x)}\right] = e^{\lim\limits_{x\to a} f(x)} = e^F$$

$$\lim_{x\to a} [\log_b f(x)] = \log_b \left[\lim_{x\to a} f(x)\right] = \log_b F\,, \quad \text{where } F > 0\,.$$

$$\lim_{x\to\pm\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\pm\infty} \frac{1}{x^n} = 0\,, \quad \text{where } n > 0\,.$$

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty$$

$$\sin x < x < \tan x\,, \quad \text{where } 0 < x < \frac{\pi}{2}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1$$

$$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{x}{\tan x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0 \qquad \left(\tfrac{1-\cos x}{x} = \tfrac{1-\cos x}{x}\cdot\tfrac{1+\cos x}{1+\cos x} = \tfrac{\sin^2 x}{x(1+\cos x)} = \tfrac{\sin x}{x}\,\tfrac{\sin x}{1+\cos x}\right)$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \qquad \left(\tfrac{1-\cos x}{x^2} = \tfrac{1-\cos x}{x^2}\cdot\tfrac{1+\cos x}{1+\cos x} = \tfrac{\sin^2 x}{x^2(1+\cos x)} = \tfrac{\sin^2 x}{x^2}\,\tfrac{1}{1+\cos x}\right)$$