First Principle and Formulae

First Principle: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Notations:

as a limit: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

as a 'fraction' : $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$, where $\delta y = y_2 - y_1 = f(x_2) - f(x_1)$, and $\delta x = x_2 - x_1$.

Important: While $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \frac{dy}{dx}\Big|_{x=c}$, $f'(c) \neq \frac{d[f(c)]}{dx}$ as $\frac{d[f(c)]}{dx} = 0$

Formulae:

$f'(kx) = kf'(x)$, where k is a constant.

$f'(x^n) = nx^{n-1}$

$[f(x) + g(x)]' = f'(x) + g'(x)$

$[f(x) \cdot g(x)]' = g(x) \cdot f'(x) + f(x) \cdot g'(x)$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\big[f\big(g(x)\big)\big]' = f'\big(g(x)\big) \cdot g'(x)$$

$\frac{d}{dx}(ky) = k\frac{d}{dx}y$, where k is a constant.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Trigonometric Functions:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$