Vector Geometry

Vector: A quantity which has *magnitude* and *direction*

On the Argand Diagram, a vector is an arrow-head – length represents magnitude, head indicates direction.

If a complex number $a + ib$ is represented as point $P(a, b)$, then the vector can be drawn from the origin as $\overrightarrow{OP}$.

Vectors have no "position", and can be drawn from anywhere on the Argand Diagram, e.g. $\overrightarrow{AB}$.

Two vectors are equal if they are of the same *magnitude* and pointing to the same *direction*,

so $\overrightarrow{OP} = \overrightarrow{AB}$ if they are of the same length (magnitude) and gradient (direction).

In this context: $z = a + ib = r(\cos\theta + i\sin\theta) \neq 0$, and is represented by $\overrightarrow{OP}$, of which the "off-the-origin" equivalent is $\overrightarrow{AB}$.

$z_m = a_m + ib_m = r_m(\cos\theta_m + i\sin\theta_m) \neq 0$, and is represented by $\overrightarrow{OP_m}$,

of which the "off-the-origin" equivalent is represented by $\overrightarrow{A_mB_m}$, where $m = 1, 2, 3, \ldots$

Modulas and Argument:

The modulas $|z| = r = \sqrt{a^2 + b^2}$ is the length of OP .

The (principal) argument $\arg(z) = \theta = \tan^{-1}\frac{b}{a}$ (or $\pm\frac{\pi}{2}$ when $a = 0$) is the smaller angle of $\angle POX$,

where OX is the positive x-axis and the sign of $\angle POX$ is the same as the sign of b.

$\arg\left(\frac{z_1}{z_2}\right) = \theta$ if OP_2 can rotate anticlockwise less than a semi-circle to reach OP_1;

$\arg\left(\frac{z_1}{z_2}\right) = -\theta$ if OP_2 can rotate clockwise less than a semi-circle to reach OP_1;

$\arg\left(\frac{z_1}{z_2}\right) = \pi$ if O is on line segment P_1P_2 internally. ($\arg(\frac{z_1}{z_2}) = 0$ if O is on P_1P_2 externally.)

Transformation: $z_1 + z_2$ is $\overrightarrow{OP'_2}$, where $\overrightarrow{P_1P'_2} = \overrightarrow{OP_2} = z_2$.

$-z_2$ is $\overrightarrow{OP_3}$, where O is the midpoint of P_2P_3 .

$z_1 - z_2$ is $\overrightarrow{OP'_3}$, where $\overrightarrow{P_1P'_3} = \overrightarrow{OP_3} = -z_2$.

z_1z_2 is a vector obtained by *enlarging* $\overrightarrow{OP_1}$ by r_2 and rotating the vector *anticlockwise* by θ_2 .

$\dfrac{z_1}{z_2}$ is a vector obtained by *shrinking* $\overrightarrow{OP_1}$ by r_2 and rotating the vector *clockwise* by θ_2 .

Properties: Geometry Properties derivable from Complex Number Relations

A *positional* off-the-origin "arrow" $\overrightarrow{P_1P_2}$ must be represented by $z_2 - z_1$, P_2 head, P_1 tail.

$\arg\left(\frac{z_1}{z_2}\right) = 0$ or π , i.e. $z_1 = kz_2$, where k is real and $k \neq 0$ $\Rightarrow$ $A_1B_1 \parallel A_2B_2$

$\arg\left(\frac{z_1}{z_2}\right) = \pm\dfrac{\pi}{2}$, i.e. $\dfrac{z_1}{z_2} = k\,i$, where k is real and $k \neq 0$ $\Rightarrow$ $A_1B_1 \perp A_2B_2$

If A_1 and A_2 coincide at A, $\angle B_1AB_2 = \left|arg\left(\frac{z_1}{z_2}\right)\right| = \left|arg\left(\frac{z_2}{z_1}\right)\right|$.

The midpoint of P_1P_2 is $\dfrac{z_1 + z_2}{2}$.