# Euler's Formula

**Given ...**

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!} + \ldots ,$$

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{6} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots ,$$

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots .$$

(No need to proof the above.)

**Express ...**

$e^{i\theta}$ as a function of $i$, $\sin\theta$ and $\cos\theta$ in a finite series (sum of finite number of terms).

**Hence ...**

prove the De Moivre's formula:

$$(\cos\theta_1 + i\sin\theta_1) \cdot (\cos\theta_2 + i\sin\theta_2) = \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2).$$