Find $\lim_{h\to 0}\dfrac{a^h-1}{h}$ from the first principle.

First, let's prove $\lim_{h\to 0}\dfrac{e^h-1}{h}=1$

$$
\begin{aligned}
\text{Let}\quad & f(h)=\frac{e^h-1}{h}\\
& hf(h)=e^h-1\\
& e^h=1+hf(h)\\
& e=[1+hf(h)]^{\frac{1}{h}}\\
\text{But}\quad & e=\lim_{h\to 0}(1+h)^{\frac{1}{h}}\quad\text{by definition}\\
\text{So}\quad & \lim_{h\to 0}(1+h)^{\frac{1}{h}}=[1+hf(h)]^{\frac{1}{h}}\\
\text{i.e.}\quad & \lim_{h\to 0}f(h)=1\\
& \therefore \lim_{h\to 0}\frac{e^h-1}{h}=1
\end{aligned}
$$

$$
\begin{aligned}
\text{Now consider}\quad \frac{d}{dx}\left(e^x\right) &= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}\\
&= \lim_{h\to 0}\frac{e^x(e^h-1)}{h}\\
&= e^x\lim_{h\to 0}\frac{e^h-1}{h}\\
&= e^x\\
\frac{d}{dx}\left(a^x\right) &= \frac{d}{dx}e^{\ln a^x}\\
&= \frac{d}{dx}e^{x\ln a}\\
&= e^{x\ln a}\cdot\frac{d}{dx}(x\ln a)\\
&= \ln a\cdot a^x
\end{aligned}
$$

$$
\begin{aligned}
\lim_{h\to 0}\frac{a^h-1}{h} &= \frac{d}{dx}(a^x)\Big|_{x=0}\\
&= \ln a\cdot a^0\\
&= \ln a
\end{aligned}
$$