

Linear Differential Equations

First Order Linear Differential Equations:

A system of differential equations:

$$\frac{dy_1}{dt} = a_{11}y_1 + a_{12}y_2 + \dots + a_{1n}y_n$$

$$\frac{dy_2}{dt} = a_{21}y_1 + a_{22}y_2 + \dots + a_{2n}y_n$$

...

$$\frac{dy_n}{dt} = a_{n1}y_1 + a_{n2}y_2 + \dots + a_{nn}y_n .$$

can be represented in matrix:

$$\frac{d\mathbf{y}}{dt} = \mathbf{A}\mathbf{y} , \quad \text{where } \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \text{ and } A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \cdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} .$$

A trial solution: $\mathbf{y}(t) = \mathbf{v}e^{\lambda t}$, where λ is an eigenvalue of \mathbf{A} , and \mathbf{v} is its eigenvector.

$$\frac{d\mathbf{y}}{dt} = \lambda \mathbf{v}e^{\lambda t} = \mathbf{A}\mathbf{v}e^{\lambda t} = \mathbf{A}\mathbf{y} .$$