Set II:

6. Write the transforms of the solutions:

a) $\ddot{c} + 6\dot{c} + 13c = 5u(t); \quad c_0 = 1, \dot{c_0} = 4.$

$(s^2C(s) - sc_0 - \dot{c_0}) + 6(sC(s) - c_0) + 13C(s) = \frac{5}{s}.$

$C(s) = \frac{\frac{5}{s} + sc_0 + \dot{c_0} + 6c_0}{s^2 + 6s + 13} = \frac{s^2 + 10s + 5}{s(s^2 + 6s + 13)}.$

e) $\frac{d^4}{dt^4}\theta + \frac{d^3}{dt^3}\theta + 6\frac{d^2}{dt^2}\theta + 12\frac{d}{dt}\theta + \theta = (1 + \sin t)u(t)$ with zero initial conditions.

$(s^4 + s^3 + 6s^2 + 12s + 1)\Theta(s) = \frac{1}{s} + \frac{1}{s^2 + 1}.$

$\Theta(s) = \frac{s^2 + s + 1}{s(s^2 + 1)(s^4 + s^3 + 6s^2 + 12s + 1)}.$

7b) find the complete time domain solutions.

$\ddot{y} + 7\dot{y} + 12y = 2\dot{u} + u; \quad y_0 = 2, \quad \dot{y_0} = 3, \quad u_0 = 0.$

$(s^2Y(s) - sy_0 - \dot{y_0}) + 7(sY(s) - y_0) + 12Y(s) = 2s \cdot \frac{1}{s} + \frac{1}{s}.$

$Y(s) = \frac{2 + \frac{1}{s} + 2s + 3 + 14}{s^2 + 7s + 12} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4} = \frac{2s^2 + 19s + 1}{s(s+3)(s+4)}.$

$A = \frac{2s^2 + 19s + 1}{(s+3)(s+4)}\Big|_{s=0} = \frac{1}{12}, \quad B = \frac{2s^2 + 19s + 1}{s(s+4)}\Big|_{s=-3} = \frac{38}{3}, \quad C = \frac{2s^2 + 19s + 1}{s(s+3)}\Big|_{s=-4} = -\frac{43}{4}.$

$y(t) = \frac{1}{12} + \frac{38e^{-3t}}{3} - \frac{43e^{-4t}}{4}.$