

MMAN2300 2013-10-05 Assignment C

Given: $m_P=440$ g, $R=42$ mm, $H=38$ mm, $L=147$ mm, $m_R=470$ g, $\bar{I}_R=1.75$ gm².

$$\omega_{AB}=3500 \text{ rpm}=3500 \times \frac{2\pi}{60} = 366.52 \text{ rad/s.}$$

$$\omega_{BC} = \frac{-\omega_{AB} \sin \theta}{\sqrt{\left(\frac{L}{R}\right)^2 - \cos^2 \theta}} = \frac{-366.52 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}}.$$

$$\sin \phi = \frac{R}{L} \cos \theta = 0.2857 \cos \theta, \quad \cos \phi = \sqrt{1 - \left(\frac{R}{L}\right)^2 \cos^2 \theta} = \sqrt{1 - 0.08163 \cos^2 \theta}.$$

$$\alpha_{BC} = \frac{\left[1 - \left(\frac{L}{R}\right)^2\right] \cos \theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2 \theta\right]^{\frac{3}{2}}} \cdot \omega_{AB}^2 = \frac{-1.511 \times 10^6 \cos \theta}{[12.25 - \cos^2 \theta]^{\frac{3}{2}}}.$$

$$\begin{aligned} a_{C_y} &= -R\omega_{AB}^2 \left\{ \frac{\left[1 - \left(\frac{L}{R}\right)^2\right] \cos^2 \theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2 \theta\right]^{\frac{3}{2}}} + \frac{\sin^2 \theta}{\sqrt{\left(\frac{L}{R}\right)^2 - \cos^2 \theta}} + \sin \theta \right\} \\ &= -5642 \left\{ \frac{-11.25 \cos^2 \theta}{[12.25 - \cos^2 \theta]^{\frac{3}{2}}} + \frac{\sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} + \sin \theta \right\} \end{aligned}$$

c) Determine the horizontal and vertical forces acting on the connecting rod at points B and C as functions of crank angle.

Assume mass m_R is distributed onto C and B:

$$\frac{m_B}{m_C} = \frac{L-H}{H}, \quad m_R = m_B + m_C = \frac{L-H}{H} m_C + m_C = \frac{L}{H} m_C, \quad m_C = \frac{H}{L} m_R, \quad m_B = \frac{L-H}{L} m_R.$$

For C, $F_{C_x} = m_C a_{C_x} = 0$, and $F_{C_y} = m_C a_{C_y} = \frac{H}{L} m_R a_{C_y}$.

Answer: $F_{C_x} = 0$.

$$\text{Answer: } F_{C_y} = -685.5 \left\{ \frac{-11.25 \cos^2 \theta}{[12.25 - \cos^2 \theta]^{\frac{3}{2}}} + \frac{\sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} + \sin \theta \right\}.$$

Tension along the connecting rod: $T = F_{C_y} (-\mathbf{i} \tan \phi + \mathbf{j}) = F_{C_y} \left(-\mathbf{i} \frac{0.2857 \cos \theta}{\sqrt{1 - 0.08163 \cos^2 \theta}} + \mathbf{j} \right)$.

For B, $a_{B_t} = 0$, $a_{B_n} = -\omega_{AB}^2 r_{B/A} = -134.3 \times 10^3 (\mathbf{i} \cos \theta + \mathbf{j} \sin \theta)$.

$$F_B = m_B a_{B_n} - T = \frac{L-H}{L} m_R a_{B_n} - T = -4.680 \times 10^4 (\mathbf{i} \cos \theta + \mathbf{j} \sin \theta) - F_{C_y} \left(-\mathbf{i} \frac{0.2857 \cos \theta}{\sqrt{1 - 0.08163 \cos^2 \theta}} + \mathbf{j} \right).$$

$$\text{Answer: } F_{B_x} = -4.680 \times 10^4 \cos \theta - 685.5 \left\{ \frac{-11.25 \cos^2 \theta}{[12.25 - \cos^2 \theta]^{\frac{3}{2}}} + \frac{\sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} + \sin \theta \right\} \frac{0.2857 \cos \theta}{\sqrt{1 - 0.08163 \cos^2 \theta}}.$$

$$\text{Answer: } F_{B_y} = -4.680 \times 10^4 \sin \theta + 685.5 \left\{ \frac{-11.25 \cos^2 \theta}{[12.25 - \cos^2 \theta]^{\frac{3}{2}}} + \frac{\sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} + \sin \theta \right\}.$$

f) Find an expression for the kinetic energy of the connecting rod as a function of crank angle.

$$T = \frac{1}{2}m_R\bar{v}^2 + \frac{1}{2}I_C\omega_{BC}^2.$$

$$\bar{\mathbf{v}} = v_C + \omega_{BC} \times r_{D/C}, \quad I_C = \bar{I}_R + m_R r_{D/C}^2 = 1.75 \times 10^{-3} + 0.470 \times 0.109^2 = 7.334 \times 10^{-3}.$$

$$v_{C/B} = \omega_{BC} \times r_{CB} = \mathbf{k} \omega_{BC} \times L(-\mathbf{i} \sin \phi + \mathbf{j} \cos \phi) = \omega_{BC} L(-\mathbf{j} \sin \phi - \mathbf{i} \cos \phi)$$

$$= \omega_{BC} L \left(-\mathbf{j} \frac{R}{L} \sin \theta - \mathbf{i} \sqrt{1 - \left(\frac{R}{L}\right)^2 \cos^2 \theta} \right) = -\omega_{BC} \left(\mathbf{j} R \cos \theta + \mathbf{i} \sqrt{L^2 - R^2 \cos^2 \theta} \right).$$

$$v_B = v_A + \omega_{AB} \times r_{B/A} = 0 + \mathbf{k} \omega_{AB} \times R(\mathbf{i} \cos \theta + \mathbf{j} \sin \theta) = \omega_{AB} R(\mathbf{j} \cos \theta - \mathbf{i} \sin \theta).$$

$$v_C = v_B + v_{C/B} = \omega_{AB} R(\mathbf{j} \cos \theta - \mathbf{i} \sin \theta) - \omega_{BC} \left(\mathbf{j} R \cos \theta + \mathbf{i} \sqrt{L^2 - R^2 \cos^2 \theta} \right).$$

$$\therefore v_{C_x} = 0, \quad \therefore v_C = \mathbf{j} v_{C_y} = \mathbf{j}(\omega_{AB} - \omega_{BC})R \cos \theta.$$

$$r_{D/C} = (L - H)(\mathbf{i} \sin \phi - \mathbf{j} \cos \phi).$$

$$\bar{\mathbf{v}} = v_C + \omega_{BC} \times r_{D/C} = \mathbf{j}(\omega_{AB} - \omega_{BC})R \cos \theta + \mathbf{k} \omega_{BC} \times (L - H)(\mathbf{i} \sin \phi - \mathbf{j} \cos \phi)$$

$$= \mathbf{j}(\omega_{AB} - \omega_{BC})R \cos \theta + \omega_{BC}(L - H)(\mathbf{j} \sin \phi + \mathbf{i} \cos \phi).$$

$$\bar{v}^2 = [\omega_{BC}(L - H) \cos \phi]^2 + [(\omega_{AB} - \omega_{BC})R \cos \theta + \omega_{BC}(L - H) \sin \phi]^2$$

$$= \left[\frac{-366.52 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \cdot 0.109 \cdot \sqrt{1 - 0.08163 \cos^2 \theta} \right]^2$$

$$+ \left[\left(366.52 + \frac{366.52 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \right) \cdot 0.042 \cos \theta - \frac{366.52 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \cdot 0.109 \cdot 0.2857 \sin \theta \right]^2$$

$$= \left[\frac{-39.95 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \sqrt{1 - 0.08163 \cos^2 \theta} \right]^2 + \left[15.39 \left(1 + \frac{\sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \right) \cos \theta - \frac{11.47 \sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} \right]^2$$

$$= \frac{1596 \sin^2 \theta}{12.25 - \cos^2 \theta} (1 - 0.08163 \cos^2 \theta) + \left[15.39 \cos \theta + \frac{15.39 \sin \theta \cos \theta}{\sqrt{12.25 - \cos^2 \theta}} - \frac{11.47 \sin^2 \theta}{\sqrt{12.25 - \cos^2 \theta}} \right]^2$$

$$= \frac{1596 \sin^2 \theta}{12.25 - \cos^2 \theta} (1 - 0.08163 \cos^2 \theta) + \left[15.39 \cos \theta + \frac{\sin \theta}{\sqrt{12.25 - \cos^2 \theta}} (15.39 \cos \theta - 11.47 \sin \theta) \right]^2$$

Putting all together ...

$$T = \frac{1}{2}m_R\bar{v}^2 + \frac{1}{2}I_C\omega_{BC}^2 = 0.53 \bar{v}^2 + 7.334 \times 10^{-3} \omega_{BC}^2.$$

$$= 0.53 \left\{ \frac{1596 \sin^2 \theta}{12.25 - \cos^2 \theta} (1 - 0.08163 \cos^2 \theta) + \left[15.39 \cos \theta + \frac{\sin \theta}{\sqrt{12.25 - \cos^2 \theta}} (15.39 \cos \theta - 11.47 \sin \theta) \right]^2 \right\}$$

$$+ 7.334 \times 10^{-3} \left(\frac{-366.52 \sin \theta}{\sqrt{12.25 - \cos^2 \theta}} \right)^2.$$

$$= \frac{845.9 \sin^2 \theta}{12.25 - \cos^2 \theta} (1 - 0.08163 \cos^2 \theta) + \left[8.157 \cos \theta + \frac{\sin \theta}{\sqrt{12.25 - \cos^2 \theta}} (8.157 \cos \theta - 6.079 \sin \theta) \right]^2$$

$$+ \frac{985.2 \sin^2 \theta}{12.25 - \cos^2 \theta}.$$

$$= \frac{1831 \sin^2 \theta - 69.05 \sin^2 \theta \cos^2 \theta}{12.25 - \cos^2 \theta} + \left[8.157 \cos \theta + \frac{\sin \theta}{\sqrt{12.25 - \cos^2 \theta}} (8.157 \cos \theta - 6.079 \sin \theta) \right]^2.$$