MMAN2300 2013-10-05 Assignment C

Given: m_P=440 g, R=42 mm, H=38 mm, L=147 mm, m_R=470 g, $\bar{I}_R$=1.75 gm^2.

$$\omega_{AB}=3500 \text{ rpm}=3500 \times \frac{2\pi}{60} = 366.52 \text{ rad/s}.$$

$$\frac{L}{R} = 3.5, \quad \left(\frac{L}{R}\right)^2 = 12.25, \quad \frac{R}{L} = 0.2857, \quad \left(\frac{R}{L}\right)^2 = 0.08163, \quad L-H = 0.109, \quad \frac{H}{L} = 0.2585.$$

$$\sin\phi = \frac{R}{L}\cos\theta, \qquad \cos\phi = \sqrt{1-\left(\frac{R}{L}\right)^2\cos^2\theta} = \frac{\sqrt{L^2-R^2\cos^2\theta}}{L}.$$

$$\omega_{BC} = \frac{-\omega_{AB}\sin\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}} = \frac{-R\omega_{AB}\sin\theta}{\sqrt{L^2-R^2\cos^2\theta}} = \frac{-R\omega_{AB}\sin\theta}{L\cos\phi} = \frac{-\omega_{AB}\sin\phi\sin\theta}{\cos\theta\cos\phi} = -\omega_{AB}\tan\phi\tan\theta.$$

$$\alpha_{BC} = \frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}\cdot\omega_{AB}^2 = \frac{(R^2-L^2)R\cos\theta}{(L^2-R^2\cos^2\theta)^{\frac{3}{2}}}\cdot\omega_{AB}^2 = \frac{(R^2-L^2)R\cos\theta}{L^3\cos^3\phi}\cdot\omega_{AB}^2$$
$$= \frac{(R^2-L^2)\sin\phi}{L^2\cos^3\phi}\cdot\omega_{AB}^2 = \left[\left(\frac{R}{L}\right)^2-1\right]\frac{\omega_{AB}^2\tan\phi}{\cos^2\phi}.$$

$$v_{C_y} = R\cos\theta(\omega_{AB}-\omega_{BC}).$$

$$a_{C_y} = -R\omega_{AB}^2\left\{\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}} + \frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}} + \sin\theta\right\}$$
$$= -R\omega_{AB}^2\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}} - R\omega_{AB}^2\frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}} - R\omega_{AB}^2\sin\theta$$
$$= -R\alpha_{BC}\cos\theta + R\omega_{AB}\omega_{BC}\sin\theta - R\omega_{AB}^2\sin\theta$$
$$= -R\alpha_{BC}\cos\theta + R\omega_{AB}(\omega_{BC}-\omega_{AB})\sin\theta$$
$$= -R\alpha_{BC}\cos\theta - v_{C_y}\omega_{AB}\tan\theta$$

c) Determine the horizontal and vertical forces acting on the connecting rod at points B and C as functions of crank angle.

F_B provides the centripetal force on B towards A and the force pushing the bottom end of the rob around.
F_C is the result of the force from the left wall in the **i** direction, and the weight of the piston in the $-$**j** direction.
These are the only two forces acting on the connecting rod.
$-F_{C_y} - m_P\, g = m_P\, a_{C_y}$.

$\boxed{F_{C_y} = -m_P\, a_{C_y} - m_P\, g.}$ Generally, $g + a_{C_y} > 0$. So $F_{C_y} < 0$ (downward).

The centre of gravity D divides the rob into CD and DB in the ratio of $L-H : H$.

$$r_D = \frac{H}{L}r_C + \frac{L-H}{L}r_B, \quad v_D = \frac{H}{L}v_C + \frac{L-H}{L}v_B, \quad a_D = \frac{H}{L}a_C + \frac{L-H}{L}a_B.$$

$$a_C = \mathbf{j}a_{C_y}, \quad r_B = R(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta), \quad v_B = R\omega_{AB}(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta), \quad a_B = -R\omega_{AB}^2(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta).$$

$$a_D = \frac{H}{L}\mathbf{j}a_{C_y} - \frac{L-H}{L}R\omega_{AB}^2(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = -\mathbf{i}\frac{L-H}{L}R\omega_{AB}^2\cos\theta + \mathbf{j}\left(\frac{H}{L}a_{C_y} - \frac{L-H}{L}R\omega_{AB}^2\sin\theta\right).$$

Alternatively, $a_D = a_C + \alpha_{\mathbf{BC}} \times r_{D/C} - \omega_{BC}^2 \cdot r_{D/C}$

$$= \mathbf{j} a_{C_y} + \mathbf{k}\ \alpha_{BC} \times [\mathbf{i}(L-H)\sin\phi - \mathbf{j}(L-H)\cos\phi] - \omega_{BC}^2 \cdot [\mathbf{i}(L-H)\sin\phi - \mathbf{j}(L-H)\cos\phi]$$

$$= \mathbf{j} a_{C_y} + \alpha_{BC} \cdot [\mathbf{j}(L-H)\sin\phi + \mathbf{i}(L-H)\cos\phi] - \omega_{BC}^2 \cdot [\mathbf{i}(L-H)\sin\phi - \mathbf{j}(L-H)\cos\phi].$$

$$= \mathbf{j} a_{C_y} + \alpha_{BC}\mathbf{j}(L-H)\sin\phi + \alpha_{BC}\mathbf{i}(L-H)\cos\phi - \omega_{BC}^2\mathbf{i}(L-H)\sin\phi + \omega_{BC}^2\mathbf{j}(L-H)\cos\phi.$$

$$= \mathbf{i}\left[\alpha_{BC}(L-H)\cos\phi - \omega_{BC}^2(L-H)\sin\phi\right] + \mathbf{j}\left[a_{C_y} + \alpha_{BC}(L-H)\sin\phi + \omega_{BC}^2(L-H)\cos\phi\right].$$

This can be proven to be equivalent to a_D above.

$$F_{B_y} + F_{C_y} - m_R\ g = m_R\ a_{D_y}.$$

$$F_{B_y} = m_R\ a_{D_y} + m_R\ g - F_{C_y} = m_R\ a_{D_y} + m_R\ g + m_P\ g + m_P\ a_{C_y}$$

$$= m_R\left[a_{C_y} + \alpha_{BC}(L-H)\sin\phi + \omega_{BC}^2(L-H)\cos\phi\right] + m_R\ g + m_P\ g + m_P\ a_{C_y}$$

$$= (m_R + m_P)(a_{C_y} + g) + m_R\left[\alpha_{BC}(L-H)\sin\phi + \omega_{BC}^2(L-H)\cos\phi\right]$$

$$= (m_R + m_P)(a_{C_y} + g) + m_R\left[\frac{(R^2 - L^2)\sin\phi}{L^2\cos^3\phi} \cdot \omega_{AB}^2 \cdot (L-H)\sin\phi + \frac{R^2\omega_{AB}^2\sin^2\theta}{L^2\cos^2\phi}(L-H)\cos\phi\right]$$

$$= (m_R + m_P)(a_{C_y} + g) + \frac{m_R\omega_{AB}^2(L-H)}{L^2\cos^2\phi}\left[(R^2 - L^2)\frac{\sin^2\phi}{\cos\phi} + R^2\sin^2\theta\cos\phi\right]$$

$$= (m_R + m_P)(a_{C_y} + g) + \frac{m_R\omega_{AB}^2(L-H)}{L^2\cos^2\phi}\left[(R^2 - L^2)\frac{\sin^2\phi}{\cos\phi} + (R^2 - L^2\sin^2\phi)\cos\phi\right]$$

$$= (m_R + m_P)(a_{C_y} + g) + \frac{m_R\omega_{AB}^2(L-H)}{L^2\cos^3\phi}\left[R^2\sin^2\phi - L^2\sin^2\phi + R^2\cos^2\phi - L^2\sin^2\phi\cos\phi^2\right]$$

$$= (m_R + m_P)(a_{C_y} + g) + \frac{m_R\omega_{AB}^2(L-H)}{L^2\cos^3\phi}\left[R^2 - L^2\sin^2\phi(1 + \cos\phi^2)\right]$$

FOLLOWINGS ARE UNFINISHED

$$= m_R\ \left(-\frac{H}{L}R\omega_{AB}^2\left\{\frac{\left[1 - \left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2\theta\right]^{\frac{3}{2}}} + \frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2 - \cos^2\theta}} + \sin\theta\right\} - \frac{L-H}{L}R\omega_{AB}^2\sin\theta\right) + (m_R + m_P)g$$

$$= -m_R\frac{H}{L}R\omega_{AB}^2\ \left\{\frac{\left[1 - \left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2\theta\right]^{\frac{3}{2}}} + \frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2 - \cos^2\theta}} + \sin\theta + \frac{L-H}{H}\sin\theta\right\} + (m_R + m_P)g$$

$$\boxed{F_{B_y} = -685.5\ \left\{\frac{-11.25\cos^2\theta}{[12.25 - \cos^2\theta]^{\frac{3}{2}}} + \frac{\sin^2\theta}{\sqrt{12.25 - \cos^2\theta}} + 3.868\sin\theta\right\} + 8.927.}$$

Moment about B: $\quad -L\sin\phi \cdot F_{C_y} - L\cos\phi \cdot F_{C_x} = \left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC}$.

$$F_{C_x} = \frac{\left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC} + L\sin\phi \cdot F_{C_y}}{-L\cos\phi} = \frac{1}{-L\cos\phi}\left[\left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC} - L\frac{R}{L}\cos\theta \cdot m_P\ g\right],$$

$$= \frac{-1}{\sqrt{L^2 - R^2\cos\theta}}\left[\left(\bar{I}_R + m_R\ H^2\right) \cdot \frac{\left[1 - \left(\frac{L}{R}\right)^2\right]\cos\theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2\theta\right]^{\frac{3}{2}}} \cdot \omega_{AB}^2 - R\cos\theta \cdot m_P\ g\right]$$

$$= \frac{-R\cos\theta}{\sqrt{L^2 - R^2\cos\theta}}\left[\left(\bar{I}_R + m_R\ H^2\right) \cdot \frac{\left(R^2 - L^2\right)}{\left(L^2 - R^2\cos^2\theta\right)^{\frac{3}{2}}} \cdot \omega_{AB}^2 - m_P\ g\right]$$

$$= \frac{-R\left(\bar{I}_R + m_R\ H^2\right) \cdot \left(R^2 - L^2\right)\omega_{AB}^2}{\left(L^2 - R^2\cos^2\theta\right)^2} \cdot \cos\theta + \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta$$

$$\boxed{F_{C_x} = \frac{582.3}{(1 - 0.08163\cos^2\theta)^2} \cdot \cos\theta + \frac{1.233}{\sqrt{1 - 0.08163\cos^2\theta}} \cdot \cos\theta.}$$

Horizontal force: $\quad F_{C_x} + F_{B_x} = m_R\ a_{D_x}$.

$$F_{B_x} = m_R\ a_{D_x} - F_{C_x} = -m_R\frac{L-H}{L}R\omega_{AB}^2\cos\theta + \frac{R\left(\bar{I}_R + m_R\ H^2\right) \cdot \left(R^2 - L^2\right)\omega_{AB}^2}{\left(L^2 - R^2\cos^2\theta\right)^2} \cdot \cos\theta - \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta$$

$$= R\omega_{AB}^2\left[-m_R\frac{L-H}{L} + \frac{\left(\bar{I}_R + m_R\ H^2\right) \cdot \left(R^2 - L^2\right)}{\left(L^2 - R^2\cos^2\theta\right)^2}\right]\cos\theta - \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta.$$

$$\boxed{F_{B_x} = -1966\cos\theta - \frac{582.3}{(1 - 0.08163\cos^2\theta)^2}\cos\theta - \frac{1.233}{\sqrt{1 - 0.08163\cos^2\theta}}\cos\theta.}$$