MMAN2300 2013-10-05 Assignment C

Given: m_P=440 g, R=42 mm, H=38 mm, L=147 mm, m_R=470 g, $\bar{I}_R$=1.75 gm^2.

ω_{AB}=3500 rpm=3500 $\times \dfrac{2\pi}{60}$ = 366.52 rad/s.

$$\frac{L}{R}=3.5,\quad \left(\frac{L}{R}\right)^2=12.25,\quad \frac{R}{L}=0.2857,\quad \left(\frac{R}{L}\right)^2=0.08163,\quad L-H=0.109,\quad \frac{H}{L}=0.2585.$$

$$\omega_{BC}=\frac{-\omega_{AB}\sin\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}=\frac{-366.52\sin\theta}{\sqrt{12.25-\cos^2\theta}},\quad \omega_{BC}^2=\frac{1.343\times10^5\sin\theta}{12.25-\cos^2\theta}.$$

$$\sin\phi=\frac{R}{L}\cos\theta=0.2857\cos\theta,\quad \cos\phi=\sqrt{1-\left(\frac{R}{L}\right)^2\cos^2\theta}=\sqrt{1-0.08163\cos^2\theta}.$$

$$\alpha_{BC}=\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}\cdot\omega_{AB}^2=\frac{-1.511\times10^6\cos\theta}{[12.25-\cos^2\theta]^{\frac{3}{2}}}.$$

$$a_{C_y}=-R\omega_{AB}^2\left\{\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}+\sin\theta\right\}$$

$$=-5642\left\{\frac{-11.25\cos^2\theta}{[12.25-\cos^2\theta]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{12.25-\cos^2\theta}}+\sin\theta\right\}$$

c) Determine the horizontal and vertical forces acting on the connecting rod at points B and C as functions of crank angle.

Let's assume that that crank shaft is driving the connecting rob and there is no pressure on top of the piston.
F_B provides the centripetal force on B towards A and the force pushing the rob upwards.
F_C is the result of the horizontal force from the (left) wall and the vertical force of the piston's weight.
These are the only two forces acting on the connecting rod. (The piston is considered an external system.

$$\boxed{F_{C_y}=-m_P\,g.}$$

The centre of gravity D divides the rob into CD and DB in the ratio of $L-H:H$.

$$r_D=\frac{H}{L}r_C+\frac{L-H}{L}r_B,\quad v_D=\frac{H}{L}v_C+\frac{L-H}{L}v_B,\quad a_D=\frac{H}{L}a_C+\frac{L-H}{L}a_B.$$

$$a_C=\mathbf{j}a_{C_y},\quad r_B=R(\mathbf{i}\cos\theta+\mathbf{j}\sin\theta),\quad v_B=R\omega_{AB}(-\mathbf{i}\sin\theta+\mathbf{j}\cos\theta),\quad a_B=-R\omega_{AB}^2(\mathbf{i}\cos\theta+\mathbf{j}\sin\theta).$$

$$a_D=\frac{H}{L}\mathbf{j}a_{C_y}-\frac{L-H}{L}R\omega_{AB}^2(\mathbf{i}\cos\theta+\mathbf{j}\sin\theta)=-\mathbf{i}\frac{L-H}{L}R\omega_{AB}^2\cos\theta+\mathbf{j}\left(\frac{H}{L}a_{C_y}-\frac{L-H}{L}R\omega_{AB}^2\sin\theta\right)$$

$$=\mathbf{i}(-0.03114\,\omega_{AB}^2\cos\theta)+\mathbf{j}(0.2585\,a_{C_y}-0.03114\,\omega_{AB}^2\sin\theta).$$

$$F_{B_y}+F_{C_y}-m_R\,g=m_R\,a_{D_y},\quad F_{B_y}=m_R\,a_{D_y}+m_R\,g+m_P\,g=m_R\left(\frac{H}{L}a_{C_y}-\frac{L-H}{L}R\omega_{AB}^2\sin\theta\right)+(m_R+m_P)g$$

$$=m_R\left(-\frac{H}{L}R\omega_{AB}^2\left\{\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}+\sin\theta\right\}-\frac{L-H}{L}R\omega_{AB}^2\sin\theta\right)+(m_R+m_P)g$$

$$=-m_R\frac{H}{L}R\omega_{AB}^2\left\{\frac{\left[1-\left(\frac{L}{R}\right)^2\right]\cos^2\theta}{\left[\left(\frac{L}{R}\right)^2-\cos^2\theta\right]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{\left(\frac{L}{R}\right)^2-\cos^2\theta}}+\sin\theta+\frac{L-H}{H}\sin\theta\right\}+(m_R+m_P)g$$

$$\boxed{F_{B_y}=-685.5\left\{\frac{-11.25\cos^2\theta}{[12.25-\cos^2\theta]^{\frac{3}{2}}}+\frac{\sin^2\theta}{\sqrt{12.25-\cos^2\theta}}+3.868\sin\theta\right\}+8.927.}$$

Moment about B: $\quad -L\sin\phi \cdot F_{C_y} - L\cos\phi \cdot F_{C_x} = \left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC}$.

$$F_{C_x} = \frac{\left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC} + L\sin\phi \cdot F_{C_y}}{-L\cos\phi} = \frac{1}{-L\cos\phi}\left[\left(\bar{I}_R + m_R\ H^2\right)\alpha_{BC} - L\frac{R}{L}\cos\theta \cdot m_P\ g\right],$$

$$= \frac{-1}{\sqrt{L^2 - R^2\cos\theta}}\left[\left(\bar{I}_R + m_R\ H^2\right)\cdot \frac{\left[1 - \left(\frac{L}{R}\right)^2\right]\cos\theta}{\left[\left(\frac{L}{R}\right)^2 - \cos^2\theta\right]^{\frac{3}{2}}}\cdot \omega_{AB}^2 - R\cos\theta \cdot m_P\ g\right]$$

$$= \frac{-R\cos\theta}{\sqrt{L^2 - R^2\cos\theta}}\left[\left(\bar{I}_R + m_R\ H^2\right)\cdot \frac{\left(R^2 - L^2\right)}{\left(L^2 - R^2\cos^2\theta\right)^{\frac{3}{2}}}\cdot \omega_{AB}^2 - m_P\ g\right]$$

$$= \frac{-R\left(\bar{I}_R + m_R\ H^2\right)\cdot\left(R^2 - L^2\right)\omega_{AB}^2}{\left(L^2 - R^2\cos^2\theta\right)^2}\cdot\cos\theta + \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta$$

$$\boxed{F_{C_x} = \frac{582.3}{(1 - 0.08163\cos^2\theta)^2}\cdot\cos\theta + \frac{1.233}{\sqrt{1 - 0.08163\cos^2\theta}}\cdot\cos\theta.}$$

Horizontal force: $\quad F_{C_x} + F_{B_x} = m_R\ a_{D_x}$.

$$F_{B_x} = m_R\ a_{D_x} - F_{C_x} = -m_R\frac{L - H}{L}R\omega_{AB}^2\cos\theta + \frac{R\left(\bar{I}_R + m_R\ H^2\right)\cdot\left(R^2 - L^2\right)\omega_{AB}^2}{\left(L^2 - R^2\cos^2\theta\right)^2}\cdot\cos\theta - \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta$$

$$= R\omega_{AB}^2\left[-m_R\frac{L - H}{L} + \frac{\left(\bar{I}_R + m_R\ H^2\right)\cdot\left(R^2 - L^2\right)}{\left(L^2 - R^2\cos^2\theta\right)^2}\right]\cos\theta - \frac{R\ m_P\ g}{\sqrt{L^2 - R^2\cos\theta}}\cos\theta.$$

$$\boxed{F_{B_x} = -1966\cos\theta - \frac{582.3}{(1 - 0.08163\cos^2\theta)^2}\cos\theta - \frac{1.233}{\sqrt{1 - 0.08163\cos^2\theta}}\cos\theta.}$$