MMAN1300 2012-10-15 AQ1

Given: m=12.6 Kg, v=5.8 m/s, M=4.2 Kg, r=1.3m

For the block, $T - mg = ma, \quad T = ma + mg \ldots (1)$

For the bar, there is T on the left, A on the right and the weight Mg in the middle.

The acceleration of the bar is $\frac{-a}{2}$ at the centre of gravity (the mid-point).

$T + A_y - Mg = M\frac{-a}{2}, \quad A_y = M\left(g - \frac{a}{2}\right) - T.$

Sub (1) into the above: $A_y = Mg - \frac{Ma}{2} - (ma + mg),$

$A_y = (M - m)g - \left(\frac{M}{2} + m\right) a \ldots (2)$

There is also a rotation on the bar about A with an anti-clockwise $\alpha = \frac{a}{r}$.

The moment of the bar about A is $I = \frac{Mr^2}{3}$.

$\tau = Mg \cdot \frac{r}{2} - Tr = I\alpha = \frac{Mr^2}{3} \cdot \frac{a}{r}.$

$\times \frac{6}{r}: \quad 3Mg - 6T = 2Ma$

Sub (1) into the above: $3Mg - 6(ma + mg) = 2Ma.$

$(2M + 6m)a = (3M - 6m)g, \quad a = \frac{3M - 6m}{2M + 6m} g \approx -7.350 \text{ ms}^{-2} \ldots (3).$

Sub (3) into (2): $A_y = (M - m)g - \left(\frac{M}{2} + m\right) \cdot \left(\frac{3M - 6m}{2M + 6m} g\right) = 25.725N$

For A_x, it can be determined by a_x of the bar at its centre of gravity, the mid-point.

At the tip, $v_t = -v, \quad v_t = -\omega r.$ At the mid-point, $v_m = -\omega \cdot \frac{r}{2} = \frac{v_t}{2} = -\frac{v}{2},$

$v_x = v_m \sin(-\theta) = \frac{v}{2} \sin\theta.$ (θ positive for anti-clockwise; x points to the right.)

$a_x = \frac{d\, v_x}{dt} = \frac{v}{2} \cos\theta \cdot \frac{d\theta}{dt} = \frac{v}{2} \cos\theta \cdot \omega = \frac{v}{2} \cos\theta \cdot \frac{v}{r} = \frac{v^2}{2r}. \qquad (\theta = 0.)$

$A_x = Ma_x = \frac{Mv^2}{2r} \approx 54.342N.$ (Towards the right into the wall)

$A = \sqrt{A_x^2 + A_y^2} \approx 60.123N.$

Alternative Method:

Given: m=12.6 Kg, v=5.8 m/s, M=4.2 Kg, r=1.3m

Total energy = PE(Block) + KE(Block) + PE(Bar) + KE(Bar) + RE(Bar).

$E = mgh + \frac{1}{2}mv^2 + MgH + \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2.$

The centre of gravity of the bar is in its mid-point. θ is positive for anti-clockwise.

$H = \frac{r}{2}\sin(-\theta), \quad V = \frac{r(-\theta)}{2}, \quad I = \frac{Mr^2}{3}, \quad \omega = \frac{v}{r}.$

$E = mgh + \frac{1}{2}mv^2 - Mg\frac{r\sin(\theta)}{2} + \frac{1}{2}M\left(\frac{r\theta}{2}\right)^2 + \frac{1}{2}\cdot\frac{Mr^2\omega^2}{3}$

$$\begin{aligned}
\frac{d}{dt}E &= mg\frac{dh}{dt} + \frac{m}{2}\cdot\frac{dv^2}{dt} - \frac{Mgr}{2}\cdot\frac{d\sin(\theta)}{dt} + \frac{Mr^2}{8}\cdot\frac{d\theta^2}{dt} + \frac{M}{6}\cdot\frac{dv^2}{dt} \\
&= mgv + \frac{m}{2}\cdot 2v\frac{dv}{dt} - \frac{Mgr\cos(\theta)}{2}\cdot\frac{d\theta}{dt} + \frac{Mr^2}{8}\cdot 2\theta\frac{d\theta}{dt} + \frac{M}{6}\cdot 2v\frac{dv}{dt} \\
&= mgv + mva - \frac{1}{2}Mgr\cos(\theta)\cdot\omega + \frac{1}{4}Mr^2\theta\cdot\omega + \frac{1}{3}Mv\cdot a \\
&= mgv + mva - \frac{1}{2}Mgv\cos(\theta) + \frac{1}{4}Mr\theta\cdot v + \frac{1}{3}Mv\cdot a \\
&= 0
\end{aligned}$$

$mg + ma - \frac{1}{2}Mg\cos(\theta) + \frac{1}{4}Mr\theta + \frac{1}{3}Ma = 0.$

When $\theta = 0, \quad mg + ma - \frac{1}{2}Mg + \frac{1}{3}Ma = 0, \quad 6mg + 6ma - 3Mg + 2Ma = 0,$

$a = \frac{3M - 6m}{2M + 6m}g \approx -7.350 \text{ ms}^{-2}.$

$-a$ is the tangential acceleration of the bar at the loose end.

Since we chose an anti-clockwise direction, $\quad -a = r\,(-\alpha), \quad a = r\,\alpha.$

At the centre of gravity of the bar (mid-point),

tangential acceleration $\quad a_T = \alpha\cdot\frac{r}{2} = \frac{a}{2} \approx -3.675 \text{ ms}^{-2}$ (downward decelerating), and

normal acceleration $\quad a_N = \frac{r}{2}\omega^2 = \frac{r}{2}\left(\frac{v}{r}\right)^2 = \frac{v^2}{2r} \approx 12.938 \text{ ms}^{-2}$ (leftward acceleration).

$T - mg = ma, \quad T = ma + mg.$

Downward force: $Mg - T - A_y = Ma_T, \quad A_y = Mg - (ma + mg) - \frac{Ma}{2} \approx 25.725 \text{ ms}^{-2}.$

$A_x = Ma_N = \frac{Mv^2}{2r} \approx 54.342 \text{ ms}^{-2}.$

$A = \sqrt{A_x^2 + A_y^2} \approx 60.123 \text{ N}.$