UNSW MATH2089 Sample 1 Q3

Q3 Concrete is vulnerable to shock vibrations, which may cause hidden damage to the material. In a study of vibration phenomena, an experiment is carried out and data is reported: including the variables **ppv** – peak particle velocity (mm/sec), and **Ratio** – ratio of ultrasonic pulse velocity after impact to that before impact in concrete prisms. Investigators fit the simple linear linear regression model:

$\mathbf{Ratio} = \beta_{\mathbf{1}} + \beta_{\mathbf{2}} \times \mathbf{ppv} + \epsilon \qquad (*)$
Note: The question uses β_1 and β_2, instead of β_0 and β_1 as in the lecture.

Using the regression analysis output (which includes residual plots and a fitted line plot) at the end of the question, answer the following questions.

a) i) Write the null and alternative hypotheses to test whether the variable **ppv** is significant in predicting the variable **Ratio**.

The null hypotheses $H_0 : \beta_2 = 0$ against
The alternative hypothese $H_a : \beta_2 \leq 0$.

ii) Carry out the test at the 1% significance level.

$\alpha = 0.01, \quad 1 - \dfrac{\alpha}{2} = 0.995, \quad$ and $t_{n-2,1-\frac{\alpha}{2}} = t_{28,0.995} = 2.763.$
(Use "upper tail probability" $p = \frac{\alpha}{2} = 0.005$, or "probability" $C = 1 - \alpha = 0.99$.)

Reject if (the estimate of β_1) $\hat{b_1} \notin \left[-t_{28,0.995} \cdot \dfrac{S}{\sqrt{S_{xx}}}, \quad t_{28,0.995} \cdot \dfrac{S}{\sqrt{S_{xx}}}\right]$
i.e. $|\hat{b_1}| > \left| t_{28,0.995} \cdot \dfrac{S}{\sqrt{S_{xx}}} \right|$.
Given the test statistic $t_0 = \sqrt{S_{xx}}\dfrac{\hat{b_1}}{S} = -7.80,$ the rejection criterion becomes
$|\hat{b_1}| > \left| t_{28,0.995} \cdot \dfrac{\hat{b_1}}{t_0} \right|$. i.e. $|t_0| > t_{28,0.995}$.
From the data $7.8 > 2.763$, so H_0 is rejected.