UNSW MATH2089 2013 Q4c

Q4c An engineer was given a project to compare the abrasive wear of two difference laminated materials, X and Y: Twelve pieces of randomly selected material X were tested, by exposing each piece to a machine measuring wear. Similarly ten randomly selected pieces of material Y were tested. The following summary gives the average wear and the standard deviation of each material:

For material X: $\bar{x} = 85$; $s_x = 4$; $n_x = 12$.
For material Y: $\bar{y} = 82$; $s_y = 5$; $n_y = 10$.

i) What assumption(s) does the engineer need to make in order to construct a valid confidence interval?

1. X and Y are independent.
2. X and Y are normally distributed.
3. The samples are random.
4. The sample sizes are large enough.

ii) Assuming that X and Y have the same population variance, find the pooled estimate of this variance.

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} = \frac{(12-1)4^2 + (10-1)5^2}{12 + 10 - 2} = 20.05 \quad \ldots \text{variance of the population.}$$

iii) Find a 99% confidence interval for the difference of the average wear of materials X and Y.

Degree of freedom $\nu = n_x + n_y - 2 = 12 + 10 - 2 = 20$. $t_{20,0.99} = 2.845$.

$$t_{20,0.99} \cdot s_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}} = 2.845 \times \sqrt{20.05} \times \sqrt{\frac{1}{12} + \frac{1}{10}} = 5.4546$$

$$\bar{x} - \bar{y} \in \left[-t_{20,0.99} \cdot s_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}},\ \ t_{20,0.99} \cdot s_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \right] = [-5.4546, 5, 4546].$$

By the way $\ldots$ $t_0 = \dfrac{\bar{x} - \bar{y}}{s_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} = \dfrac{3}{\sqrt{20.05} \times \sqrt{\frac{1}{12} + \frac{1}{10}}} = 1.5647$. $|t_0| \not> t_{20,0.99} \ldots$ accept.