MECH326 Stress and Strain

Stress: (1-9 p.15)

Stress is a state property at a specific point within a body, which is a function of load, geometry, temperature, and manufacturing processing. Stresses are discussed mostly in the context of external forces (load-induced) but other stresses (e.g. thermal stresses) are discussed in more advanced context. When the source of stress is removed, the stress will return to zero, unless it has reached a certain limit (strength).

Stress is a vector, and therefore directional. E.g. normal stress σ, shear stress τ, and normal stress σ_1.

Strength: (1-9 p.15)

Strengths are the magnitudes of stresses at which something of interest occurs (usually loss of functions), such as the proportional limit yielding, or fracture (usually when loss of functions occurs). Depending on this "thing of interest", there are different types of strengths: yield strength S_y, ultimate strength S_u, shear yield strength S_{sy} and endurance strength S_e etc.

Strength is an inherent property of a part. The geometry of the part has an effect on the stress it can take. This follow that different point of a part may have different strength.

Strength is a scaler, but as it is associated with the stress vector, even at the same point, strength may be higher in one direction but lower in another.

Uncertainty: (1-10 p.16)

The strength is measured *nominally* in the lab with an uncertainty. The stress in the field (where the part is used) is also measured (or predicted) with an uncertainty.

For example, the maximum load of the passenger lift in a building may vary by δ_σ, say 15% (because people's weight cannot be predicted precisely). The "maximum allowable load" σ as shown in the lift needs to be δ_σ lower than the *real* maximum load that the lift can take (σ_o).

$\sigma = \sigma_o - \sigma_o\delta_\sigma, \quad \sigma_o = \sigma \times \dfrac{1}{1-\delta_\sigma}.$ (σ needs to be increased by the loss-of-function parameter $\dfrac{1}{1-\delta_\sigma}$, or $\dfrac{1}{0.85}$.)

On the other hand, the lift manufacturer has measured the strength of the lift and found it to be S, say 20000 N, with an uncertainty of δ_S, say 20%. So S may be δ_S higher than its *real* strength S_o.

$S = S_o + S_o\delta_S, \quad S_o = S \times \dfrac{1}{1+\delta_S}.$ (S needs to be decreased by the maximum allowable parameter $\dfrac{1}{1+\delta_S}$, or $\dfrac{1}{1.2}$.)

Let us define the design factor $n_d \equiv \dfrac{S}{\sigma}$.

The *loss-of-function strength* must be stronger than the *maximum allowable load* by at least n_d times.

For safety reasons, we must assume the worst and have the *real* load always under the *real* strength.

$\because \quad \sigma_o < S_o, \quad \therefore \quad \sigma \times$ loss-of-function-parameter $< S \times$ maximum-allowable-parameter,

$$n_d = \frac{S}{\sigma} > \frac{\text{loss-of-function-parameter}}{\text{maximum-allowable-parameter}}.$$

Dimension and Tolerances 1-13 p.19

$x_l < x < x_u$, x normal size; $x_u - x_l$ tolerance;

Mating parts: r_{int}, r_{ext} internal and external member radius;

$r_{ext} - r_{int} > 0$ radial clearance; $r_{int} - r_{ext} > 0$ radial interference;

Allowance: minimum clearance or maximum interference.

Tensile and Compressive Strengths

Stress $\sigma = \dfrac{P}{A}$ where P is load and A is cross section (perpendicular to P).

Within the *proportional limit* , $\sigma = E\epsilon$, where E is the *Young's Modulus.* ...the Hooke's Law.

$$\epsilon = \frac{\sigma}{E} = \frac{P}{AE}. \quad d\delta = \epsilon\ dx, \quad \delta = \int_0^L \epsilon\ dx = \int_0^L \frac{P}{AE}\ dx = \frac{PL}{AE}.$$

Different points may have different stress. The stress concentration factor $K = \dfrac{\sigma_{max}}{\sigma_{avg}}$, $\sigma_{max} = K\dfrac{P}{A}$.

Strain $\epsilon = \dfrac{l - l_0}{l_0}$. A *stress-strain* diagram has strain as the horizontal axis and stress vertical.

Over a range, $d\epsilon = \dfrac{dl}{l}$, $\epsilon = \displaystyle\int_{l_0}^{l} \frac{dl}{l} = ln\frac{l}{l_0}$.

The material will restore before the *elastic limit.* Beyond that, it is plastic (permanently set even after the load is removed).

Beyond the *yield point,* the strain will increase rapidly without corresponding increase in stress. The *ultimate* strength correspond to the maximum stress, from which the strain will increase even regardless of the stress (which is generally decreasing as it cannot hold the load anymore).

When a part is under vertical tensile stress, the Poisson's Ratio $\nu = \dfrac{-\epsilon_{trans}}{\epsilon_{axial}}$ indicates the necking effect. where $\epsilon_{trans} < 0$ is the stress perpendicular to the tension, and $\epsilon_{axial} > 0$ parallel.

Torsional Stress

The shear stress τ of a bar of length l_0 at a point ρ from the axis is related to the twisted angle θ by:

$\tau = \dfrac{G\rho}{l_0}\theta$. At the outer radius r, the maximum shear stress $\tau_{max} = \dfrac{Gr}{l_0}\theta$.

G is the material stiffness property called *shear modulus* or *the modulus of rigidity.*

$\tau = \left(\dfrac{\rho}{r}\right)\tau_{max}$. The unit of τ is Pa (or Nm^{-2}), same as G.

An imaginary line drawn parallel to the axis but ρ away from it will deviate by an angle γ after twisting.

$\gamma = \dfrac{\rho\theta}{l_0}$ when $l_0 \gg \rho$. $\tau = \gamma G$. $\gamma_{max} = \dfrac{r\theta}{l_0}$, $\gamma = \left(\dfrac{\rho}{r}\right)\gamma_{max}$.

Second Moment of Area (or Polar Moment of Area): $J = \displaystyle\int_A \rho^2 dA$. For a circle, $J = \displaystyle\int_0^r \rho^2(2\pi\rho\ d\rho) = \frac{\pi r^4}{2}$.

Shear force at ρ: $dV = \tau dA = \dfrac{\rho}{r}\tau_{max}\ dA$. Torque $dT = \rho\ dV$, $T = \displaystyle\int_A \rho\ dV = \int_A \rho \cdot \frac{\rho}{r}\tau_{max}\ dA = J\frac{\tau_{max}}{r}$.

$$\boxed{\tau_{max} = \frac{Tr}{J}.} \quad \text{Generally, } \tau = \frac{T\rho}{J}. \quad \gamma = \frac{\tau}{G} = \frac{\rho\theta}{l_0}, \quad \theta = \frac{\tau l_0}{\rho G} = \frac{T\rho}{J}\cdot\frac{l_0}{\rho G}. \quad \boxed{\theta = \frac{Tl_0}{JG}.} \quad \frac{T}{J} = \frac{G\theta}{l_0} = \frac{\tau}{\rho}.$$