MECH326 Chapter 2 – Materials

2-1 Material Strength and Stiffness

True stress-strain diagrams take into account of reduced area in $\sigma = \dfrac{P}{A}$, instead of using the original area A_0 as in *engineering* (or *nominal*) stress-strain diagrams. True strain $\epsilon = \displaystyle\int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$ … Eq 2-4 p.34

Also, for constant volume part, $\epsilon = \ln \dfrac{A_0}{A}$ … Eq 2-17 p.40.

Shear stress $\tau = \dfrac{G\rho}{l_0}\theta$ where $\rho(< r)$ is the distance from the centre, l_0 is the gauge length.

$\tau_{max} = \dfrac{Gr}{l_0}\theta = \dfrac{Tr}{J}$ (max at outer edge).

$S_{su} = \dfrac{T_u r}{J}$ … Eq 2-7 p.35 the modulus of rupture (at the outer edge), NOT the ultimate torsional strength.

The energy in the process is the area under the stress-strain curve: $U \cong \displaystyle\int_{l_0}^{l} \sigma A \, dl$. As $l = l_0(1+\epsilon)$, $dl = l_0 \, d\epsilon$.

$U = A l_0 \displaystyle\int_0^{\epsilon} \sigma \, d\epsilon = Vu$, where V is the volume and $u(\epsilon)$ is the energy 'density', assuming constant A (before necking).

(Note: The unit of u is Jm^{-3}= Nm^{-2}= Pa.)

Modulus of resilience: energy absorbed per unit volume at yielding point: $u_R = u(\epsilon_y) \cong \displaystyle\int_0^{\epsilon_y} \sigma \, d\epsilon$ (ignore necking).

$u_R \cong \dfrac{1}{2} S_y \epsilon_y = \dfrac{1}{2} S_y (S_y/E) = \dfrac{S_y^2}{2E}$ … Eq 2-9 p.36.

Modulus of toughness: $u_T = u(\epsilon_f) \cong \left(\dfrac{S_y + S_{ut}}{2}\right)\epsilon_f$ … Eq 2-11 p.36.

2-3 Strength and Cold Work

Ductility (Table A-20 p.1040): $R = 1 - \dfrac{A_f}{A_0}$ where A_0 & A_f is the cross sectional area originally and at fracture respectively.

Datsko (cold working): $\sigma = \sigma_0 \epsilon^m$ where σ is true stress, σ_0 is strain-strengthening coefficient, ϵ is true plastic strain, and m is strain-strengthening exponent. $m = \epsilon_u$.