Q18) Find tangent of

$$y = x^2 + x^3 \text{ at } (x_0, y_0).$$

$$\frac{dy}{dx} = 2x + 3x^2$$

$$\begin{aligned}
\text{Tangent:}\quad y &= (2x_0 + 3x_0^2)(x - x_0) + y_0 \\
&= (2x_0 + 3x_0^2)x - (2x_0 + 3x_0^2)x_0 + x_0^2 + x_0^3 \\
&= (2x_0 + 3x_0^2)x - 2x_0^2 - 3x_0^3 + x_0^2 + x_0^3 \\
&= (2x_0 + 3x_0^2)x - x_0^2 - 2x_0^3
\end{aligned}$$

i) Show the tangent cuts the curve at $x = -(1 + 2x_0)$.

Let (x_1, y_1) be a point on the tangent with $x_1 = -(1 + 2x_0)$

On the tangent, when $x = x_1$

$$\begin{aligned}
y_1 &= (2x_0 + 3x_0^2)x_1 - x_0^2 - 2x_0^3 \\
&= (2x_0 + 3x_0^2)x_1 - x_0^2(1 + 2x_0) \\
&= (2x_0 + 3x_0^2)x_1 + x_0^2 x_1 \\
&= (2x_0 + 3x_0^2 + x_0^2)x_1 \\
&= (2x_0 + 4x_0^2)x_1 \\
&= -2x_0[-(1 + 2x_0)]x_1 \\
&= -2\left(-\frac{1 + x_1}{2}\right)[x_1]\, x_1 \qquad \text{Note:}\quad x_0 = -\frac{1 + x_1}{2} \\
&= (1 + x_1)x_1^2 \\
&= x_1^2 + x_1^3
\end{aligned}$$

$\therefore (x_1, y_1)$ is on the curve as well.

ie The tangent cuts the curve at $x = -(1 + 2x_0)$.

ii) Find the tangent which cuts the curve at $(0, 0)$.

From i) $x_1 = 0$, $y_1 = 0$ and $x_0 = -\frac{1 + x_1}{2} = -\frac{1}{2}$

$$\begin{aligned}
\text{Tangent:}\quad y &= (2x_0 + 3x_0^2)x - x_0^2 - 2x_0^3 \\
&= (-1 + \frac{3}{4})x - \frac{1}{4} + \frac{1}{4}
\end{aligned}$$

$x + 4y = 0$ ... tangent that cut the curve at $(0, 0)$