

Q19) Tangent of $y = -3 + 4x - x^2$ passing through the origin touch the parabola at (x_0, y_0) .

i) Show that $y + 3 - 4x_0 + x_0^2 = (4 - 2x_0)(x - x_0)$ is a tangent at (x_0, y_0) .

$$\frac{dy}{dx} = 4 - 2x \quad \text{so the tangent is } y - y_0 = (4 - 2x_0)(x - x_0).$$

$$(x_0, y_0) \text{ is on the curve, so } y_0 = -3 + 4x_0 - x_0^2.$$

$\therefore y + 3 - 4x_0 + x_0^2 = (4 - 2x_0)(x - x_0)$ is a tangent.

ii) Show that $x_0 = \pm\sqrt{3}$.

Since the tangent passes through the origin, $(0, 0)$ is on it.

$$0 + 3 - 4x_0 + x_0^2 = (4 - 2x_0)(0 - x_0)$$

$$3 - 4x_0 + x_0^2 + 4x_0 - 2x_0^2 = 0$$

$$x_0^2 = 3$$

$$x_0 = \pm\sqrt{3}$$

iii) Show that the tangent at ii) is $y = (4 \pm 2\sqrt{3})x$.

$$\therefore y + 3 - 4x_0 + x_0^2 = (4 - 2x_0)(x - x_0)$$

$$y + 3 \mp 4\sqrt{3} + 3 = (4 \mp 2\sqrt{3})(x \mp \sqrt{3})$$

$$y + 6 \mp 4\sqrt{3} = (4 \mp 2\sqrt{3})x \mp (4 \mp 2\sqrt{3})\sqrt{3}$$

$$y + 6 \mp 4\sqrt{3} = (4 \mp 2\sqrt{3})x + 6 \mp 4\sqrt{3}$$

$$\therefore y = (4 \mp 2\sqrt{3})x$$

$$\text{ie } y = (4 \pm 2\sqrt{3})x$$