i) Show that $\int_0^{\frac{\pi}{4}} \frac{1-\sin 2x}{1+\sin 2x}\, dx = \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

$$\begin{aligned}
LHS &= \int_0^{\frac{\pi}{4}} \frac{1-\sin 2\left(\frac{\pi}{4}-x\right)}{1+\sin 2\left(\frac{\pi}{4}-x\right)}\, dx \\
&= \int_0^{\frac{\pi}{4}} \frac{1-\sin\left(\frac{\pi}{2}-2x\right)}{1+\sin\left(\frac{\pi}{2}-2x\right)}\, dx \\
&= \int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{1+\cos 2x}\, dx \\
&= \int_0^{\frac{\pi}{4}} \frac{2\sin^2 x}{2\cos^2 x}\, dx \\
&= \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \\
&= RHS
\end{aligned}$$

ii) Hence find $\int_0^{\frac{\pi}{4}} \frac{1-\sin 2x}{1+\sin 2x}\, dx$

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \qquad \text{from i)} \\
&= \Big[\tan x - x \Big]_0^{\frac{\pi}{4}} \\
&= \tan\frac{\pi}{4} - \frac{\pi}{4} \\
&= 1 - \frac{\pi}{4}
\end{aligned}$$