

Ex 1 g)

$$\begin{aligned} & \int \frac{\sin x \, dx}{2 + \sin x} \\ &= \int \frac{\frac{2t}{1+t^2}}{2 + \frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2} \, dt \\ &= \int \frac{t}{1+t^2+t} \cdot \frac{2}{1+t^2} \, dt \\ &= \int \frac{-2}{1+t^2+t} + \frac{2}{1+t^2} \, dt \\ &= \int \frac{-2}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} \, dt + 2 \int \frac{1}{1+t^2} \, dt \\ &= \int \frac{\frac{2}{\sqrt{3}}}{\left(t + \frac{1}{2}\right) + \frac{\sqrt{3}}{2}} - \frac{\frac{2}{\sqrt{3}}}{\left(t + \frac{1}{2}\right) - \frac{\sqrt{3}}{2}} \, dt + 2 \int \frac{1}{1+t^2} \, dt \\ &= \frac{2}{\sqrt{3}} \ln \left| \frac{t + \frac{1}{2} + \frac{\sqrt{3}}{2}}{t + \frac{1}{2} - \frac{\sqrt{3}}{2}} \right| + 2 \tan^{-1} t + C \\ &= \frac{2}{\sqrt{3}} \ln \left| \frac{2 \tan \frac{x}{2} + 1 + \sqrt{3}}{2 \tan \frac{x}{2} + 1 - \sqrt{3}} \right| + x + C \end{aligned}$$