

Q23

Let $u = \cos^2 x$

$$du = -2 \cos x \sin x dx$$

$$\begin{aligned} & \int \sin^3 x \cos^2 x dx \\ &= \int \sin^3 x \cos^2 x \frac{du}{-2 \cos x \sin x} \\ &= -\frac{1}{2} \int \sin^2 x \cos x du \\ &= -\frac{1}{2} \int (1 - \cos^2 x) \cos x du \\ &= -\frac{1}{2} \int (\cos x - \cos^3 x) du \\ &= -\frac{1}{2} \int (\sqrt{u} - \sqrt{u}^3) du \\ &= -\frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} \right) + C \\ &= \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C \end{aligned}$$