Ex2 (c) Show that $\int_0^\pi \frac{x\,dx}{1+\sin x} = \pi$

$$\begin{aligned}
&\int_0^\pi \frac{x\,dx}{1+\sin x} \\
&= \int_0^\pi \frac{(\pi - x)\ dx}{1+\sin(\pi - x)} \\
&= \int_0^\pi \frac{(\pi - x)\ dx}{1+\sin x} \\
&= \int_0^\pi \frac{\pi\,dx}{1+\sin x} - \int_0^\pi \frac{x\,dx}{1+\sin x} \\
\text{ie}\quad 2\int_0^\pi \frac{x\,dx}{1+\sin x} &= \int_0^\pi \frac{\pi\,dx}{1+\sin x} \\
&= \pi \int_0^\pi \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}\,dx \\
&= \pi \int_0^\pi \frac{1-\sin x}{1-\sin^2 x} dx \\
&= \pi \int_0^\pi \frac{1-\sin x}{\cos^2 x} dx \\
&= \pi \int_0^\pi \left(\sec^2 x - \sec x\ \tan x\right) dx \\
&= \pi \left(\int_0^\pi \sec^2 x\,dx - \int_0^\pi \sec x\ \tan x\,dx \right) \\
&= \pi \left(\big[\tan x\big]_0^\pi - \big[\sec x\big]_0^\pi \right) \\
&= \pi\left[(\tan\pi - \tan 0) - (\sec\pi - \sec 0)\right] \\
&= \pi\left[(0-0) - (-1-1)\right] \\
&= 2\pi \\
\therefore \int_0^\pi \frac{x\,dx}{1+\sin x} &= \pi
\end{aligned}$$