

Ex1 (g) Show that $\int_{-2}^2 x^2 (2-x)^6 dx = \int_0^4 x^6 (2-x)^2 dx = \frac{11}{63} \cdot 2^{15}$

Let $y = 2 - x$

$$dy = -dx$$

$$\begin{aligned} & \int_{-2}^2 x^2 (2-x)^6 dx \\ &= - \int_{2+2}^{2-2} (2-y)^2 y^6 dy \\ &= \int_0^4 y^6 (2-y)^2 dy \\ &= \int_0^4 x^6 (2-x)^2 dx \end{aligned}$$

$$\therefore \int_{-2}^2 x^2 (2-x)^6 dx = \int_0^4 x^6 (2-x)^2 dx$$

$$\begin{aligned} & \int_0^4 x^6 (2-x)^2 dx \\ &= \int_0^4 4x^6 - 4x^7 + x^8 dx \\ &= \left[\frac{4x^7}{7} - \frac{x^8}{2} + \frac{x^9}{9} \right]_0^4 \\ &= \left[\frac{72x^7 - 63x^8 + 14x^9}{126} \right]_0^4 \\ &= \frac{36 \cdot 2^{15} - 126 \cdot 2^{15} + 112 \cdot 2^{15}}{126} \\ &= \frac{22 \cdot 2^{15}}{126} \\ &= \frac{11}{63} \cdot 2^{15} \end{aligned}$$