Ex1 (g) Show that $\int_{-2}^{2} x^2 (2-x)^6 \, dx = \int_0^4 x^6 (2-x)^2 \, dx = \frac{11}{63} \cdot 2^{15}$

$$
\begin{aligned}
&\int_{-2}^{2} x^2 (2-x)^6 \, dx \\
=&\int_0^4 (x-2)^2 \left[2-(x-2)\right]^6 dx \\
=&\int_0^4 (x-2)^2 (4-x)^6 \, dx \\
=&\int_0^4 \left[(4-x)-2\right]^2 \left[4-(4-x)\right]^6 dx \\
=&\int_0^4 (2-x)^2 x^6 dx \\
=&\int_0^4 x^6 (2-x)^2 \, dx
\end{aligned}
$$

$$\therefore \int_{-2}^{2} x^2 (2-x)^6 \, dx = \int_0^4 x^6 (2-x)^2 \, dx$$

$$
\begin{aligned}
&\int_0^4 x^6 (2-x)^2 \, dx \\
=&\int_0^4 4x^6 - 4x^7 + x^8 \, dx \\
=&\left[\frac{4x^7}{7} - \frac{x^8}{2} + \frac{x^9}{9}\right]_0^4 \\
=&\left[\frac{72x^7 - 63x^8 + 14x^9}{126}\right]_0^4 \\
=&\frac{36 \cdot 2^{15} - 126 \cdot 2^{15} + 112 \cdot 2^{15}}{126} \\
=&\frac{22 \cdot 2^{15}}{126} \\
=&\frac{11}{63} \cdot 2^{15}
\end{aligned}
$$

$$\therefore \int_{-2}^{2} x^2 (2-x)^6 \, dx = \int_0^4 x^6 (2-x)^2 \, dx = \frac{11}{63} \cdot 2^{15}$$