Eg4 (b) Show that $\int_0^{\frac{\pi}{2}} \frac{\cos x+3\sin x}{\sin x+\cos x}\, dx = \pi$

$$
\begin{aligned}
&\int_0^{\frac{\pi}{2}} \frac{\cos x+3\sin x}{\sin x+\cos x}\, dx \\
=&\int_0^{\frac{\pi}{2}} \frac{\cos x+3\sin x}{\sin x+\cos x}\cdot\frac{\sin x+\cos x}{\sin x+\cos x}\, dx \\
=&\int_0^{\frac{\pi}{2}} \frac{4\sin x\cos x+3\sin^2 x+\cos^2 x}{(\sin x+\cos x)^2}\, dx \\
=&\int_0^{\frac{\pi}{2}} \frac{2\cdot 2\sin x\cos x+2\sin^2 x-1+2}{1+2\sin x\cos x}\, dx \\
=&\int_0^{\frac{\pi}{2}} \frac{2+2\sin 2x-\cos 2x}{1+\sin 2x}\, dx \\
=&\int_0^{\frac{\pi}{2}} 2\, dx-\int_0^{\frac{\pi}{2}} \frac{\cos 2x}{1+\sin 2x}\, dx \\
=&\int_0^{\frac{\pi}{2}} 2\, dx-\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{d\,(1+\sin 2x)}{1+\sin 2x} \\
=&\Big[2x\Big]_0^{\frac{\pi}{2}}-\frac{1}{2}\Big[\ln|1+\sin 2x|\Big]_0^{\frac{\pi}{2}} \\
=&\pi-\frac{1}{2}[\ln 1-\ln 1] \\
=&\pi
\end{aligned}
$$