

Eg2 (c) Show that $\int_0^\pi x \cos^2 x \, dx = \frac{\pi^2}{4}$

$$\begin{aligned}\int_0^\pi x \, dx &= \left[\frac{x^2}{2} \right]_0^\pi \\ &= \frac{\pi^2}{2} \\ \text{but } \int_0^\pi x \, dx &= \int_0^\pi x (\sin^2 x + \cos^2 x) \, dx \\ &= \int_0^\pi x \left[\cos^2 \left(\frac{\pi}{2} - x \right) + \cos^2 x \right] \, dx \\ &= \int_0^\pi x [\cos^2 x + \cos^2 x] \, dx \\ &= 2 \int_0^\pi x \cos^2 x \, dx\end{aligned}$$

$$\begin{aligned}\therefore \int_0^\pi x \cos^2 x \, dx &= \frac{1}{2} \int_0^\pi x \, dx \\ &= \frac{\pi^2}{4}\end{aligned}$$