Year 12 Chapter 6B Q5 b)

The following illustration contains many useful techniques and things to watch out. Read all the notes carefully.

$$\because \quad \cos 2x = 1 - 2\sin^2 x$$
$$\therefore \quad \sin^2 x = \frac{1-\cos 2x}{2}$$

Likewise, $\sin^2 2x = \dfrac{1-\cos 4x}{2}$ (by replacing $x$ with $2x$)

$$\begin{aligned}
I &= \int \sin^2\ 2x\ dx \\
&= \int \frac{1-\cos 4x}{2}\ dx \\
&= \int \frac{1}{2} - \frac{\cos 4x}{2}\ dx \\
&= \int \frac{1}{2}\ dx - \frac{1}{2}\int \cos 4x\ dx \\
&= \frac{1}{2}\ x - \frac{1}{2}\left(\frac{1}{4}\sin 4x\right) + C \qquad \text{(Must remove all } \int \text{ signs and add } C \text{ in the same step.)} \\
&= \frac{1}{2}\ x - \frac{1}{8}\sin 4x + C.
\end{aligned}$$

Notes:

In order to use the formula $\int \cos x\ dx = \sin x + C$, the variable in cos and the one in $d$ must be the same, so we have to make up the factor 4.

You can image $dx$ being replaced by $\dfrac{d(4x)}{4}$, so

$$\int \cos 4x\ dx = \int \cos 4x\ \frac{d(4x)}{4} = \frac{1}{4}\int \cos 4x\ d(4x) = \frac{1}{4}\sin 4x + C \quad \text{(now both cos and } d \text{ carry } 4x\text{).}$$

However, you are not allowed to write $d(4x)$, even it is correct in concept.
Therefore, in your work you need to skip the steps in between and write:

$$\int \cos 4x\ dx = \frac{1}{4}\sin 4x + C.$$

Alternatively, you can introduce a new variable $u = 4x$, so $du = 4dx$ and $dx = \dfrac{du}{4}$.

$$\int \cos 4x\ dx = \int \cos u\ \frac{du}{4} = \frac{1}{4}\int \cos u\ du \qquad \text{(now the variable in cos and the one in } d \text{ are the same)}$$
$$= \frac{1}{4}\sin u + C = \frac{1}{4}\sin 4x + C.$$

IMPORTANT:
The answer can only contain $x$, not $u$. So you need to change $u$ back to the equivalent expression of $x$.