Year 11 Chapter 7G Question 11

Q11 (a) Suppose $y = \dfrac{u}{x}$, where u is a function of x. Show that $y + x\dfrac{dy}{dx} = \dfrac{du}{dx}$.

$$
\begin{aligned}
\text{LHS} &= y + x\frac{dy}{dx} \\
&= y + x \cdot \frac{d}{dx}\left(\frac{u}{x}\right) \\
&= y + \cancel{x} \cdot \frac{x\frac{du}{dx} - u\frac{d\cancel{x}}{\cancel{dx}}}{x^{\cancel{2}}} \\
&= y + \frac{\cancel{x}}{\cancel{x}} \cdot \frac{du}{dx} - \frac{u}{x} \\
&= \cancel{y} + \frac{du}{dx} - \cancel{y} \\
&= \text{RHS}.
\end{aligned}
$$