

Given a parabola of $x = 2at$ and $y = at^2$ and a point $(2at, at^2)$:

$$\frac{dy}{dt} = 2at, \quad \frac{dx}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2at}{2a} = t$$

$$\frac{dy}{dx} = t$$

Tangent at $(2at, at^2)$ is $xt - y - at^2 = 0$

Tangents at $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ intersect at $(a(p+q), apq)$.

Normal at $(2at, at^2)$ is $x + yt - at^3 - 2at = 0$

Normals at $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ intersect at $(-apq(p+q), a(p^2 + pq + q^2 + 2))$.

$$x + yp - ap^3 - 2ap = 0 \tag{1}$$

$$x + yq - aq^3 - 2aq = 0 \tag{2}$$

$$(1) - (2) : \quad y(p - q) - a(p - q)(p^2 + pq + q^2) - 2a(p - q) = 0$$

$$y = a(p^2 + pq + q^2 + 2)$$

$$x = -yp + ap^3 + 2ap$$

$$x = -ap(p^2 + pq + q^2 + 2) + ap^3 + 2ap$$

$$x = -ap^3 - ap^2q - apq^2 - 2ap + ap^3 + 2ap$$

$$x = -apq(p + q)$$