Cone Cylinder

Q6 a ii):

Using Formulae

The cone volume formula is $V_{cone} = \pi r^2 \frac{h}{3}$, where h is the hight.

In this question $V_{cone} = \pi r^2 \frac{2x}{3}$.

The volume of the cylendar at the bottom $V_{cyl} = \pi r^2 x$.

$\because r^2 = 20^2 - (2x)^2 = 400 - 4x^2$.

$\therefore$ the total volume $V = V_{cone} + V_{cyl} = \pi r^2 \frac{2x}{3} + \pi r^2 x = \pi r^2 x \left(\frac{2}{3} + 1\right)$

$$= \frac{5}{3}\pi(400 - 4x^2)x = \frac{20}{3}\pi(100x - x^3).$$

Using Integration

In fact, the volume formulae are derived using integration anyway, so you may go straight to treat it as a graph and integrate to find the volume of the solid formed by rotating the graph about the x-axis.

Let us use the same diagram we drew in the lesson, with u as the horizontal axis, and R as the radius at u.

$\frac{R}{u} = \frac{r}{2x}$, so $R = \frac{ru}{2x}$. $R^2 = \frac{r^2}{(2x)^2}u^2$. ($r$ and x are constants. Only u is varying here.)

$$\begin{aligned}
V &= \int_0^{2x} \pi R^2 \, du + \int_{2x}^{3x} \pi r^2 \, du \\
&= \int_0^{2x} \pi \frac{r^2}{(2x)^2} u^2 \, du + \pi r^2 \int_{2x}^{3x} du \\
&= \pi \frac{r^2}{(2x)^2} \int_0^{2x} u^2 \, du + \pi r^2 \int_{2x}^{3x} du \\
&= \pi r^2 \left(\frac{1}{(2x)^2} \left[\frac{u^3}{3}\right]_0^{2x} + \left[u\right]_{2x}^{3x} \right) \\
&= \pi r^2 \left(\frac{1}{(2x)^2} \left[\frac{(2x)^3}{3}\right] + \left[3x - 2x\right] \right) \\
&= \pi r^2 \left(\frac{2x}{3} + x \right) \\
&= \frac{5}{3}\pi r^2 x \\
&= \frac{5}{3}\pi(400 - 4x^2)x \\
&= \frac{20}{3}\pi(100x - x^3).
\end{aligned}$$