

Y12 4U 2005 Q3 a iii.

(a) Ellipse E : $\frac{x^2}{100} + \frac{y^2}{75} = 1$.

(iii) A circle is tangential to the ellipse E at $P(5, 7\frac{1}{2})$ and at $Q(5, -7\frac{1}{2})$. Show that the centre of the circle is $(1\frac{1}{4}, 0)$.

Solution:

(Note: This solution is an alternative to using the result of (ii), the normals.)

Strategy: Let the centre of the circle be $(h, 0)$.

The equation of the circle C is $(x - h)^2 + y^2 = r^2$.

Differentiate both sides: $d[(x - h)^2 + y^2] = 2(x - h) dx + 2y dy = 0$, $\frac{dy}{dx} = -\frac{x - h}{y}$.

The gradient of tangent at $P(5, 7\frac{1}{2})$:

For E : $m_E = -\frac{b^2x}{a^2y} = -\frac{75 \times 5}{100 \times 7\frac{1}{2}} = -\frac{1}{2}$.

For C : $m_C = -\frac{x - h}{y} = \frac{h - 5}{7\frac{1}{2}} = \frac{2h - 10}{15}$.

$\therefore m_C = m_E$, $\therefore \frac{2h - 10}{15} = -\frac{1}{2}$, $h = 1\frac{1}{4}$.

The centre of the circle $(h, 0) = (1\frac{1}{4}, 0)$.